Parallel Algorithms

Design and Implementation

Lecture 2 – Processor oblivious algorithms

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Lecture 2

- **Remind: Work W and depth D :**
  - With work-stealing schedule:
    - #steals = O(pD)
    - Execution time on p procs = W/p + O(D) w.h.p.
    - Similar bound achieved with processors with changing speed or multiprogrammed systems.
- **How to parallelize ?**
  - 1/ There exists a fine-grain parallel algorithm that is optimal in sequential
    - Work-stealing and Communications
  - 2/ Extra work induced by parallel can be amortized
  - 3/ Work and Depth are related
    - Adaptive parallel algorithms

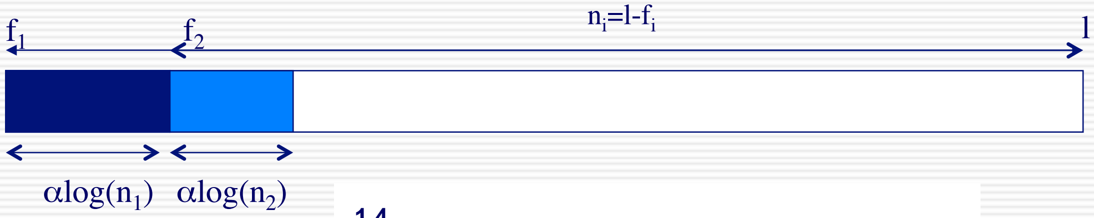
First examples

- **Put overhead on the steals :**
  - Example Accumulate

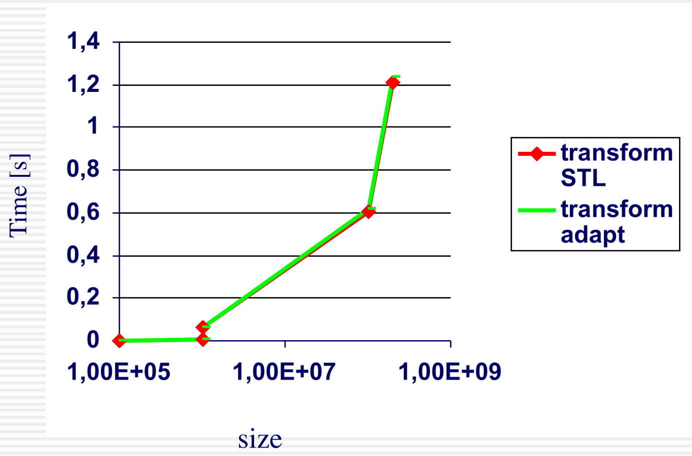
- **Follow an optimal sequential algorithm:**
  - Example: Find_if

Adaptive coupling: Amortizing synchronizations (parallel work extraction)

Example : STL **transform** STL : loop with n independent computations

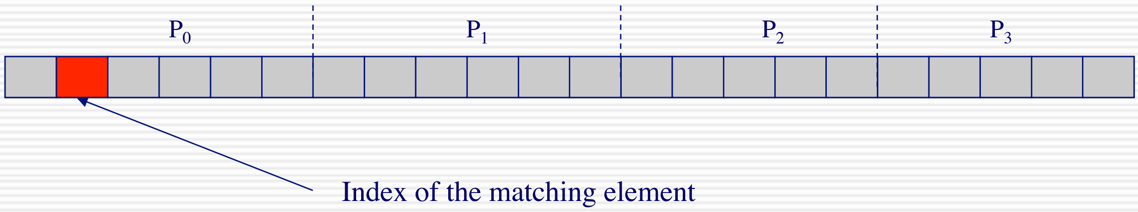


Machine :
AMD Opteron Opteron 875
2,2 Ghz,
Compiler gcc, option –O2



Amortizing Parallel Arithmetic overhead: example: find_if

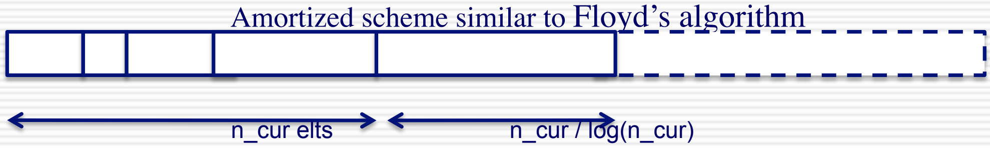
- **For some algorithms:**
  - W_{seq} unknown prior to execution
  - Worst case work W is not precise enough: we may have $W >> W_{seq}$
- **Example: find_if : returns the index of the first element that verifies a predicate.**



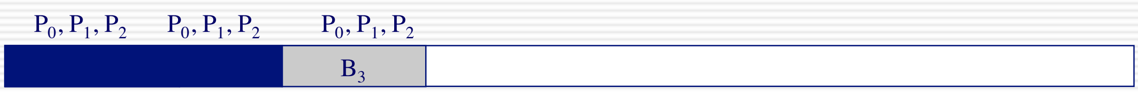
- Sequential time is $T_{seq} = 2$
- Parallel time= time of the last processor to complete: here, on 4 processors: $T_4 = 6$

Amortizing Parallel Arithmetic overhead: example: find_if

- To adapt with provable performances ($W_{par} \sim W_{seq}$) : compute in parallel no more work thant the work performed by the sequential algorithm (**Macro-loop** [Danjean, Gillard, Guelton, Roch, Roche, PASCO'07]), Amortized scheme similar to Floyd's algorithm



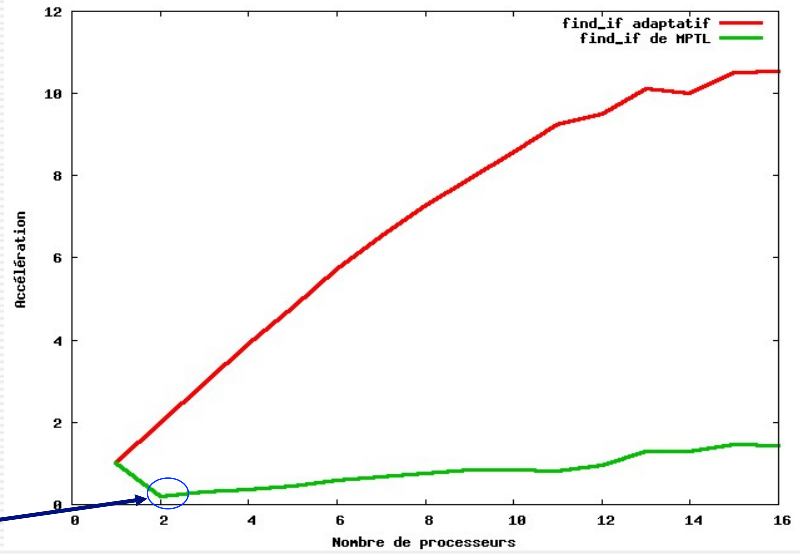
- Example : **find_if**



Amortizing Parallel Arithmetic overhead: example: find_if [Daouda Traore 2009]

- **Example : find_if STL**
  - Comparison with find_if parallel MPTL [Baertschiger 06]

Machine :
AMD Opteron (16 cœurs);
Data: doubles;
Array size: 10^6;
Position element: 10^5;
TimeSTL : 3,60 s;
Predicate time ≈ 36µ



Speed-down (speed-up < 1)

Amortizing Parallel Arithmetic overhead: example: find_if [Daouda Traore 2009]

- **Example : find_if STL**
  - Speed-up w.r.t. STL sequential tim and the position of the matching element.

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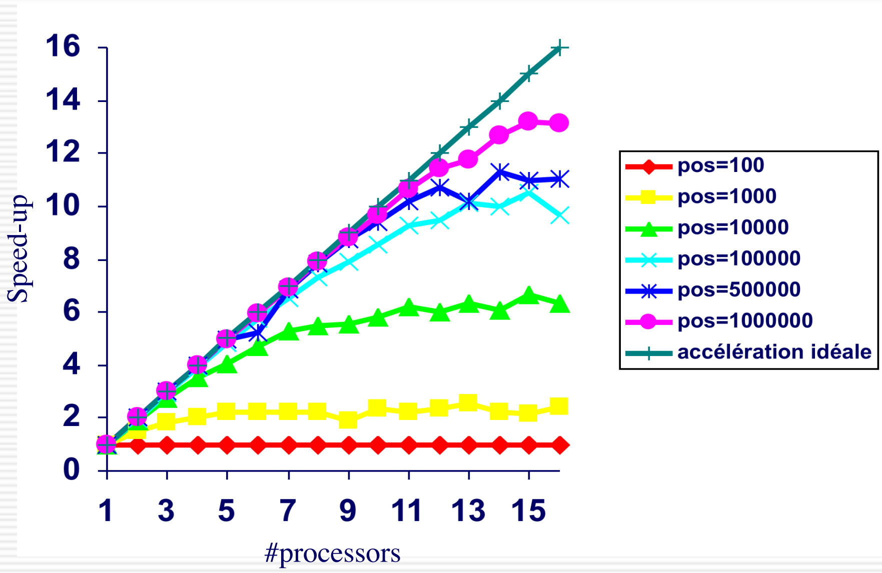


Overview

• Introduction : interactive computation, parallelism and processor oblivious
  • Overhead of parallelism : parallel prefix

• Machine model and work-stealing

• **Scheme 1: Extended work-stealing : concurently sequential and parallel**

3. Work-first principle and adaptability

- **Work-first principle**: -implicit- dynamic choice between two executions :
  - a **sequential *"depth-first"*** execution of the parallel algorithm (local, default) ;
  - a **parallel "*breadth-first*"** one.
- Choice is performed at runtime, depending on resource idleness:
  rare event if Depth is small to Work
- **WS adapts parallelism to processors with practical provable performances**
  - Processors with changing speeds / load (data, user processes, system, users,
  - Addition of resources (fault-tolerance [Cilk/Porch, Kaapi, …])

- **The choice is justified only when the sequential execution of the parallel algorithm is an efficient sequential algorithm:**
  - Parallel Divide&Conquer computations
  - …

-> **But,** this may not be general in practice

How to get both optimal work W_1 and $D = W_\infty$ small?

- General approach**: to** mix **both**
  - a **sequential** algorithm with optimal work $\boldsymbol{W_1}$
  - and a fine grain **parallel** algorithm with minimal depth $\underline{\boldsymbol{D}}$ = critical time $\boldsymbol{W_\infty}$

- Folk technique **:** *parallel, than sequential*
  - Parallel algorithm until a certain « grain »; then use the sequential one
  - Drawback : $\boldsymbol{W_\infty}$ increases ;o) …and, also, the number of steals

- *Work-preserving speed-up technique* **[Bini-Pan94]** *sequential, then parallel* **Cascading [Jaja92] :**
  Careful interplay of both algorithms to build one with both

  W_∞ **small** and W_1 **= O(** W_{seq} **)**

  - Use the work-optimal sequential algorithm to reduce the size
  - Then use the time-optimal parallel algorithm to decrease the time
  - Drawback : sequential at coarse grain and parallel at fine grain ;o(

Extended work-stealing: concurrently sequential and parallel

Based on the work-stealing and the ***Work-first*** *principle* :

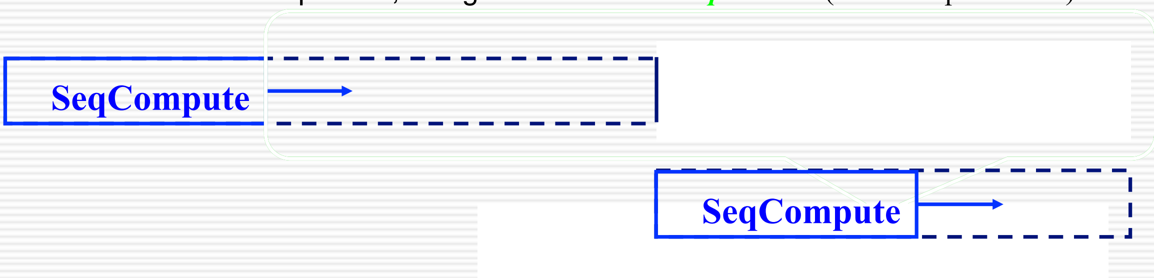
Instead of optimizing the **sequential execution** of the **best parallel** algorithm,
let optimize the **parallel execution** of the ***best sequential*** algorithm

Execute always a sequential algorithm to reduce parallelism overhead

⇒ parallel algorithm is used only if a processor becomes **idle** (ie *workstealing*) [Roch&al2005,…]
to ***extract parallelism*** from the remaining work a sequential computation

Assumption : two concurrent algorithms that are complementary:

- - one sequential : ***SeqCompute*** (always performed, the priority)
  - the other parallel, fine grain : ***LastPartComputation*** (often not performed)



Extended work-stealing : concurrently sequential and parallel

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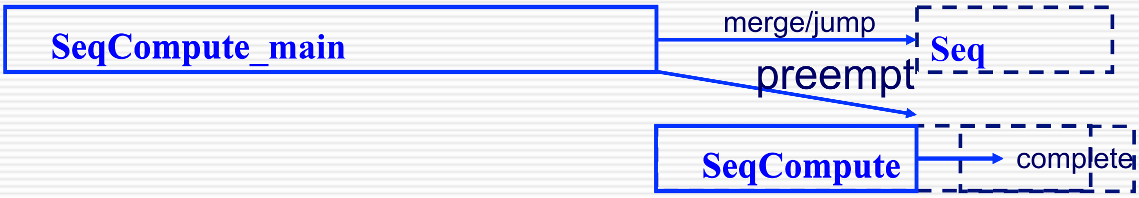

Note:

- **merge and jump** operations to ensure non-idleness of the victim
- Once *SeqCompute_main* completes, it becomes a work-stealer

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- **Scheme 2: Amortizing the overhead of synchronization (Nano-loop)**

Extended work-stealing and granularity

- **Scheme of the sequential process : nanoloop**

```
While (not completed(Wrem) ) and (next_operation hasn't been stolen)
{
    atomic { extract_next k operations ; Wrem -= k  ; }
    process the k operations extracted ;
}
```

- ***Processor-oblivious* algorithm**
  - Whatever *p* is, it performs O(*p.D*) preemption operations (« continuation faults »)
    -> ***D*** should be as small as possible to maximize both speed-up and locality
  - If no steal occurs during a (sequential) computation, then its *arithmetic work* is optimal to the one W_{opt} of the sequential algorithm (no spawn/fork/copy)
    -> ***W*** should be as close as possible to W_{opt}
- Choosing **k = Depth(W_{rem})** does not increase the depth of the parallel algorithm while ensuring ***O(W / D)*** *atomic operations* **:**
  since $D > log_2 W_{rem}$, **then if *p = 1*:** $W \sim W_{opt}$
- **Implementation :** atomicity in nano-loop based without lock
  - Efficient mutual exclusion between sequential process and parallel work-stealer
- ***Self-adaptive granularity***

Interactive application with time constraint

Anytime Algorithm:

- Can be stopped at any time (with a result)
- Result quality improves as more time is allocated

In Computer graphics, anytime algorithms are common:

Level of Detail algorithms (time budget, triangle budget, etc…)

Example: Progressive texture loading, triangle decimation (Google Earth)

Anytime processor-oblivious algorithm:

On ***p*** processors with average speed Π_{ave}, it outputs in a fixed time ***T***
a result with the same quality than
a sequential processor with speed Π_{ave} in time $p.\Pi_{ave}$.

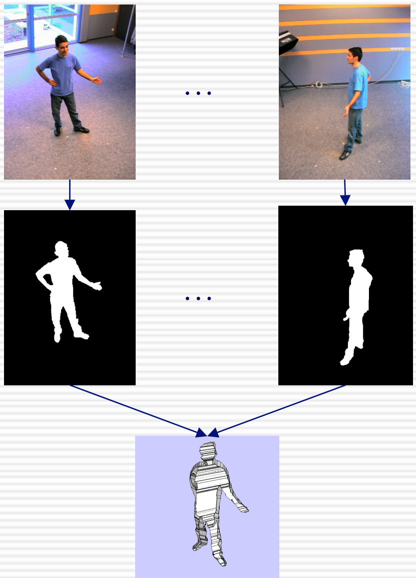
Example: Parallel Octree computation for 3D Modeling

Parallel 3D Modeling

3D Modeling :

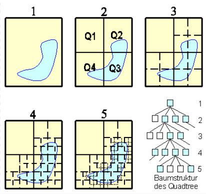
build a 3D model of a scene from a set of calibrated images

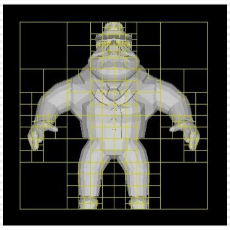
On-line 3D modeling for interactions: 3D modeling from multiple video streams (30 fps)

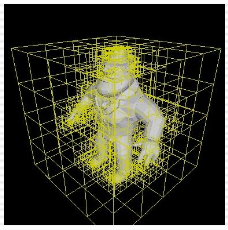


Octree Carving [L. Soares 06]

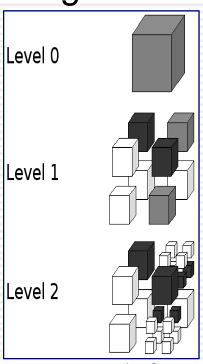
A classical recursive anytime 3D modeling algorithm.





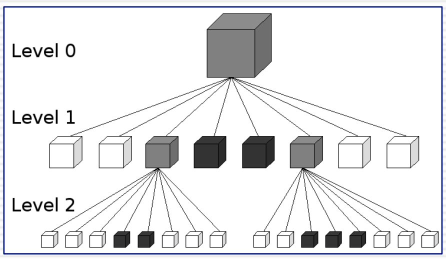


Standard algorithms with time control:



Depth first
+ iterative deepening

State of a cube:
- Grey: mixed => split
- Black: full : stop
- White: empty : stop



Width first

At termination: quick test to decide all grey cubes time control

Width first parallel octree carving

Well suited to work-stealing

-Small critical path, while huge amount of work (eg. D = 8, W = 164 000)

- non-predictable work, non predictable grain :

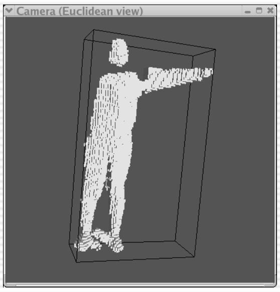
For cache locality, each level is processed by a self-adaptive grain :

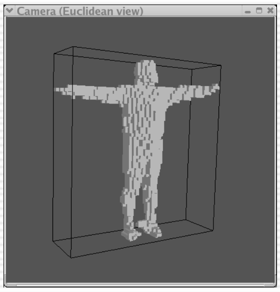
"sequential iterative" / "parallel recursive split-half"

Octree needs to be "balanced" when stopping:

- Serially computes each level *(with small overlap)*
- Time deadline (30 ms) managed by signal protocol

Unbalanced





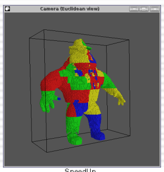
Balanced

Theorem: W.r.t the adaptive in time T on p procs., the sequential algorithm:

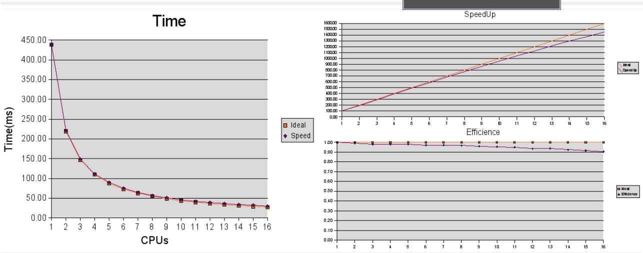
- goes at most one level deeper : $|d_s - d_p| \leq 1$;
- computes at most : $n_s \leq n_p + O(\log n_s)$.

Results

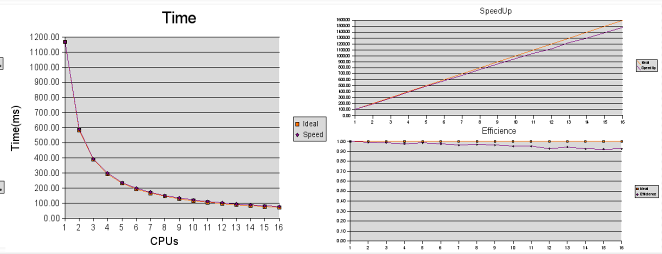
[L. Soares 06]



- 16 core Opteron machine, 64 images
- Sequential: 269 ms, 16 Cores: 24 ms
- 8 cores: about 100 steals (167 000 grey cells)



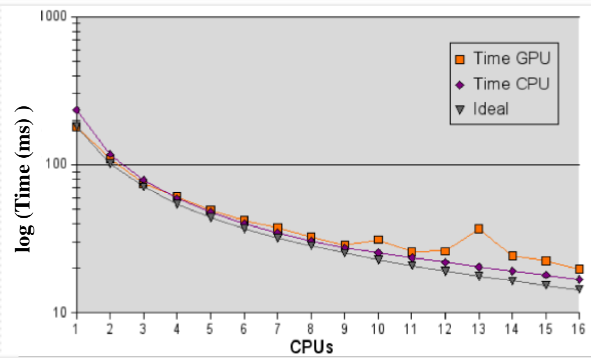
8 cameras, levels 2 to 10



64 cameras, levels 2 to 7

result: CPUs+GPU

- 1 GPU + 16 CPUs
- GPU programmed in OpenGL
- efficient coupling till 8 but does not scale



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- **Scheme 3: Amortizing the overhead of parallelism (Macro-loop)**

4. Amortizing the arithmetic overhead of parallelism

Adaptive scheme : extract_seq/nanoloop // extract_par
- ensures an optimal number of operation on 1 processor
- but no guarantee on the work performed on p processors

Eg (C++ STL): find_if (first, last, predicate)
locates the first element in [First, Last) verifying the predicate

This may be a drawback (unneeded processor usage) :
- undesirable for a library code that may be used in a complex application, with many components
- (or not fair with other users)
- increases the time of the application :
  - *any parallelism that may increase the execution time should be avoided*

Motivates the building of **work-optimal** parallel adaptive algorithm (**processor oblivious**)

4. Amortizing the arithmetic overhead of parallelism (cont'd)

Similar to nano-loop for the sequential process :
• that balances the -atomic- local work by the depth of the remaindering one

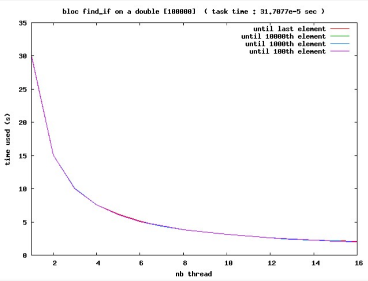
Here, by **amortizing** the work induced by the extract_par operation, ensuring this **work to be *small*** enough :
• Either w.r.t the -useful- **work already performed**
• Or with respect to the - useful - **work yet to performed** (if known)
• or both.

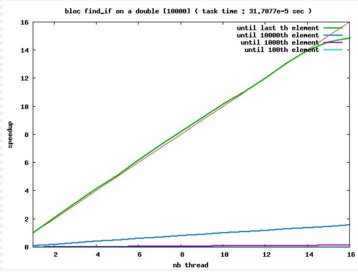
Eg : find_if (first, last, predicate) :
• only the work already performed is known (on-line)
• then prevent to assign more than $\alpha(W_{done})$ operations to work-stealers
• Choices for α(n) **:**
  • **n/2   :   similar to Floyd's iteration**   (approximation ratio = 2)
  • **n/log* n :** to ensure   optimal usage of the work-stealers

Results on find_if

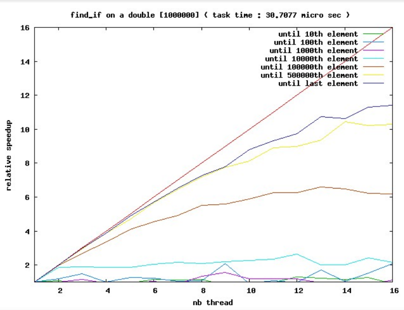
[S. Guelton]

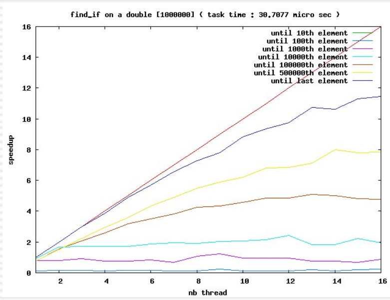
N doubles : time predicate ~ 0.31 ms





With no amortization macroloop





With amortization macroloop

5. Putting things together

processor-oblivious prefix computation

Parallel algorithm based on :

- compute-seq / extract-par scheme

- nano-loop for compute-seq

- macro-loop for extract-par

Parallelism induces overhead :

e.g. Parallel prefix on fixed architecture

- **Prefix problem** :
  - input : $a_0, a_1, \ldots, a_n$
  - output : $\pi_1, \ldots, \pi_n$ with

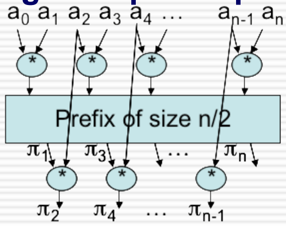
$$\pi_i = \prod_{k=0}^{i} a_k$$

- **Sequential** algorithm :
  - for ($\pi[0] = a[0]$, $i = 1$; $i <= n$; i++) $\pi[i] = \pi[i-1] * a[i]$;

performs only **n** *operations*

- **Fine grain optimal parallel** algorithm :

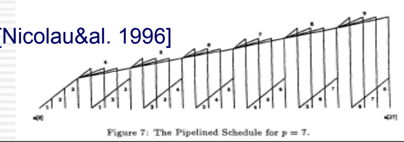
[Ladner-Fisher-81]



Critical time = 2. *log* n
but performs **2.n ops**

- Tight lower bound on **p identical processors**:

[Nicolau&al. 1996]



Optimal time T_p = 2n / (p+1)
but performs **2.n.p/(p+1) ops**

Parallel requires twice more operations than sequential !!

Lower bound(s) for the prefix

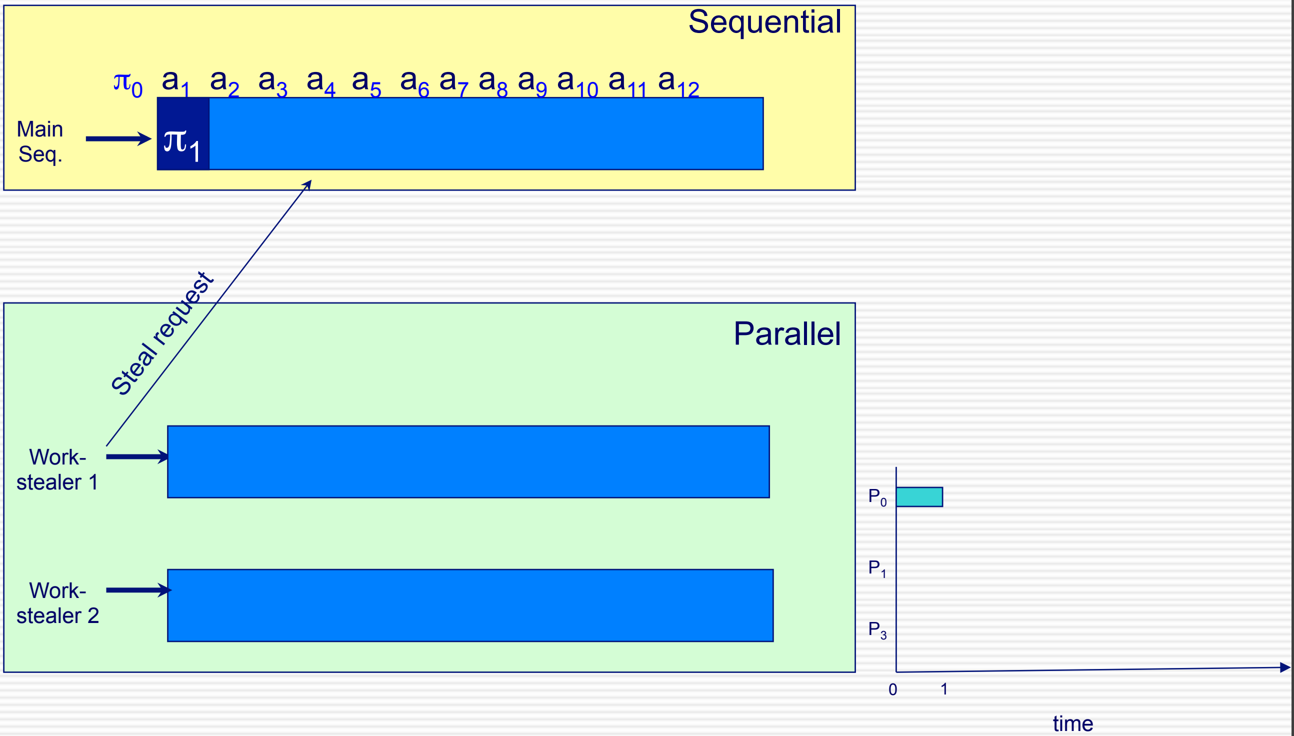
Prefix circuit of depth d

$\Downarrow$ [Fitch80]

#operations > $2n - d$

$$\boxed{\text{parallel time} \geq \frac{2n}{(p+1).\Pi_{ave}}}$$

P-Oblivious Prefix on 3 proc.



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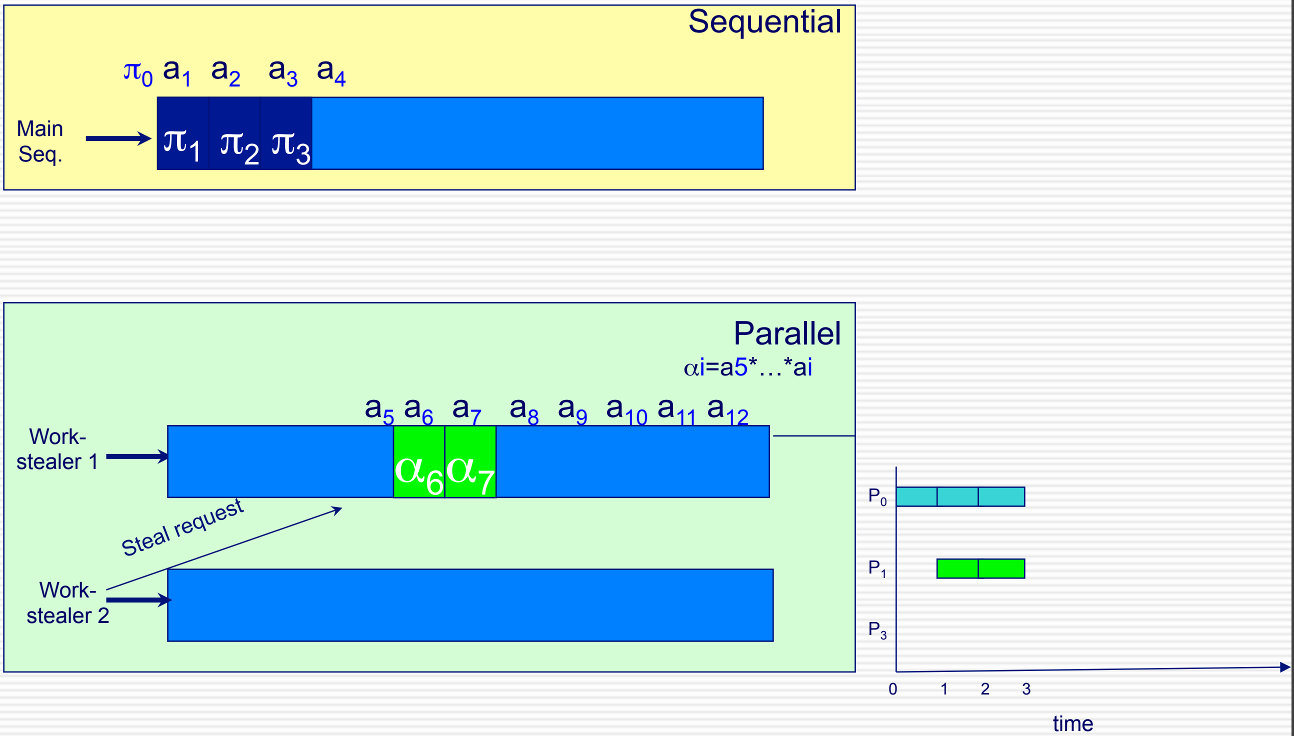

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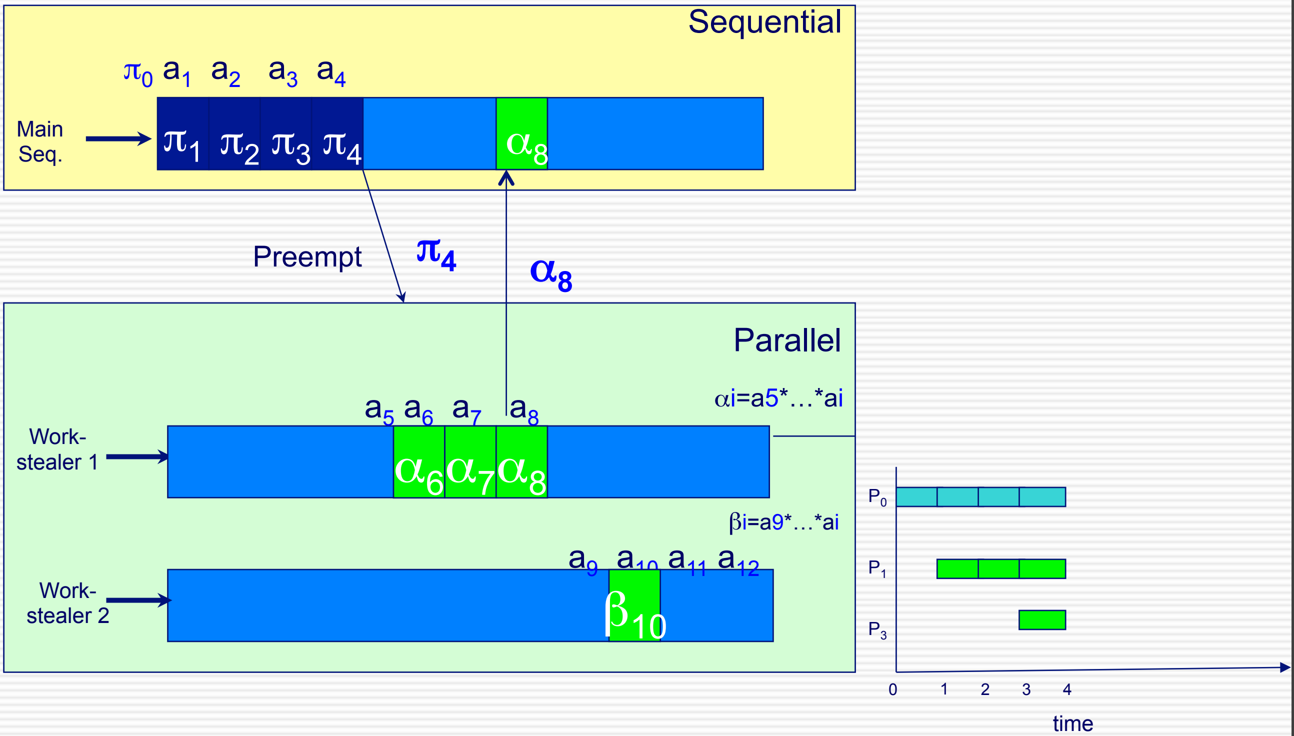

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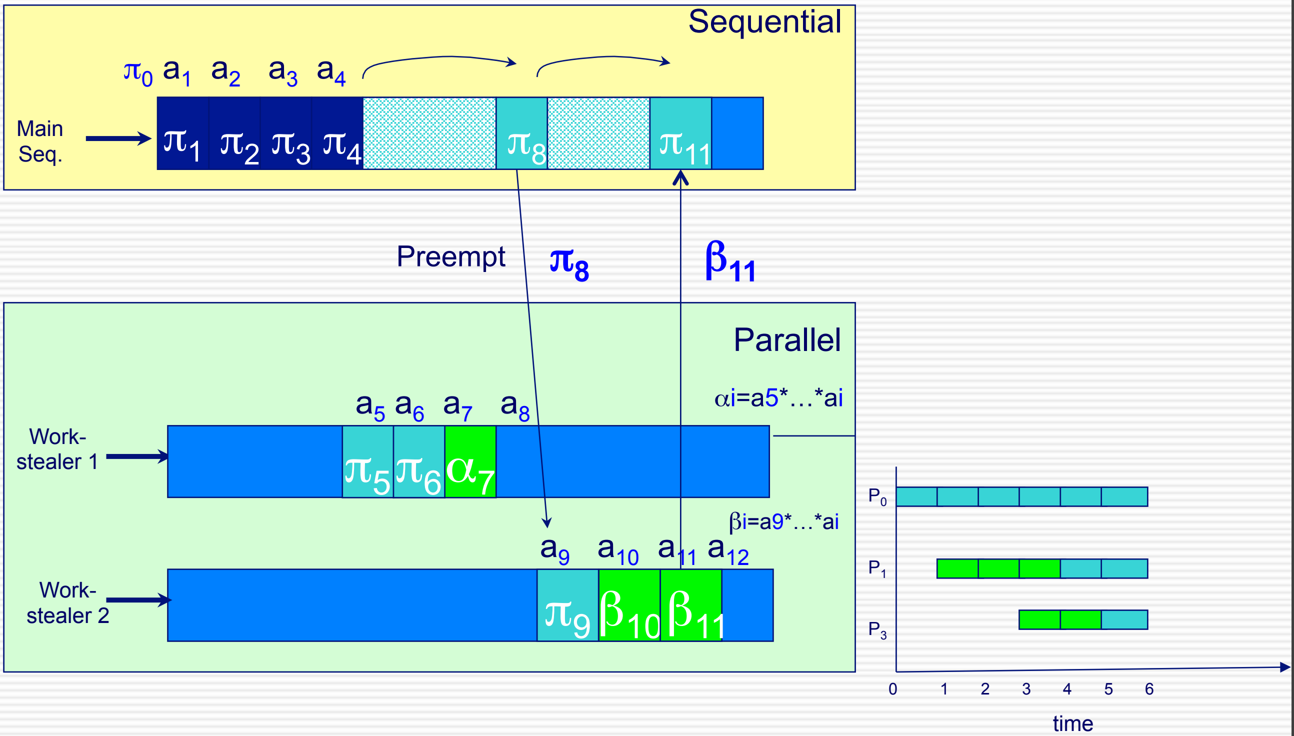

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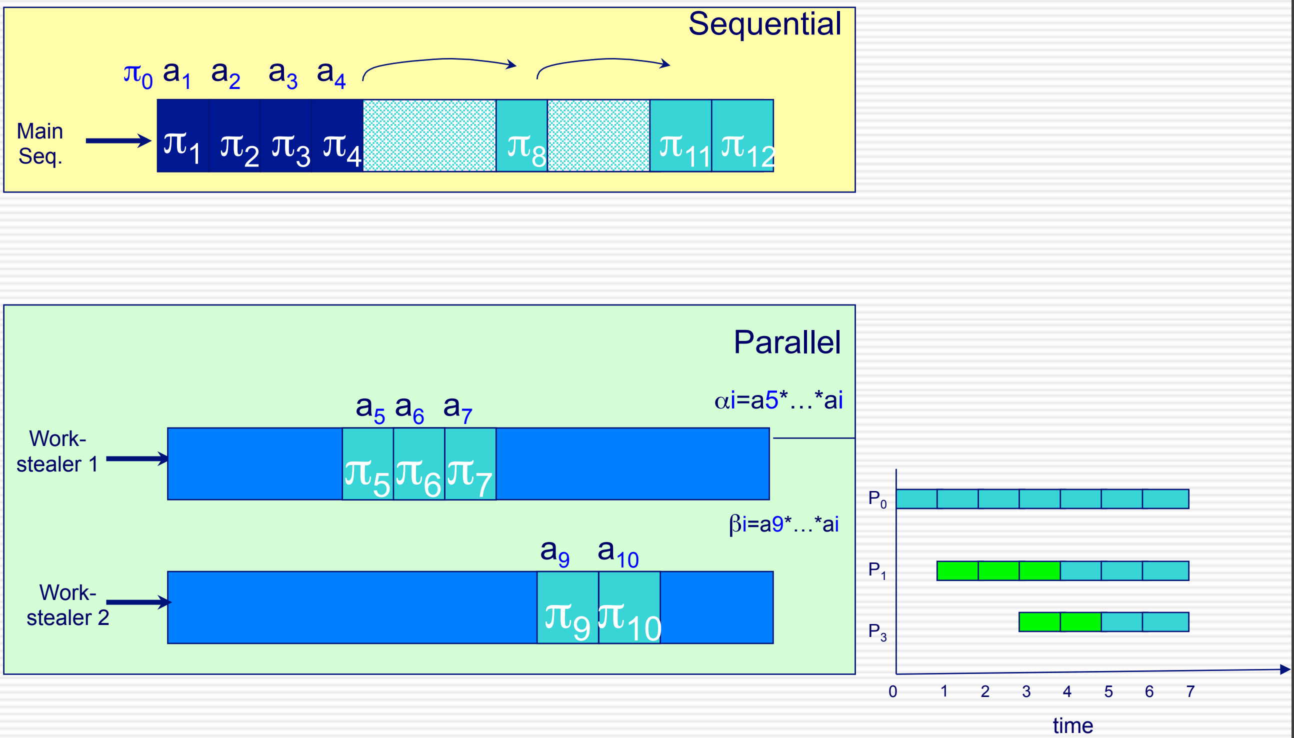

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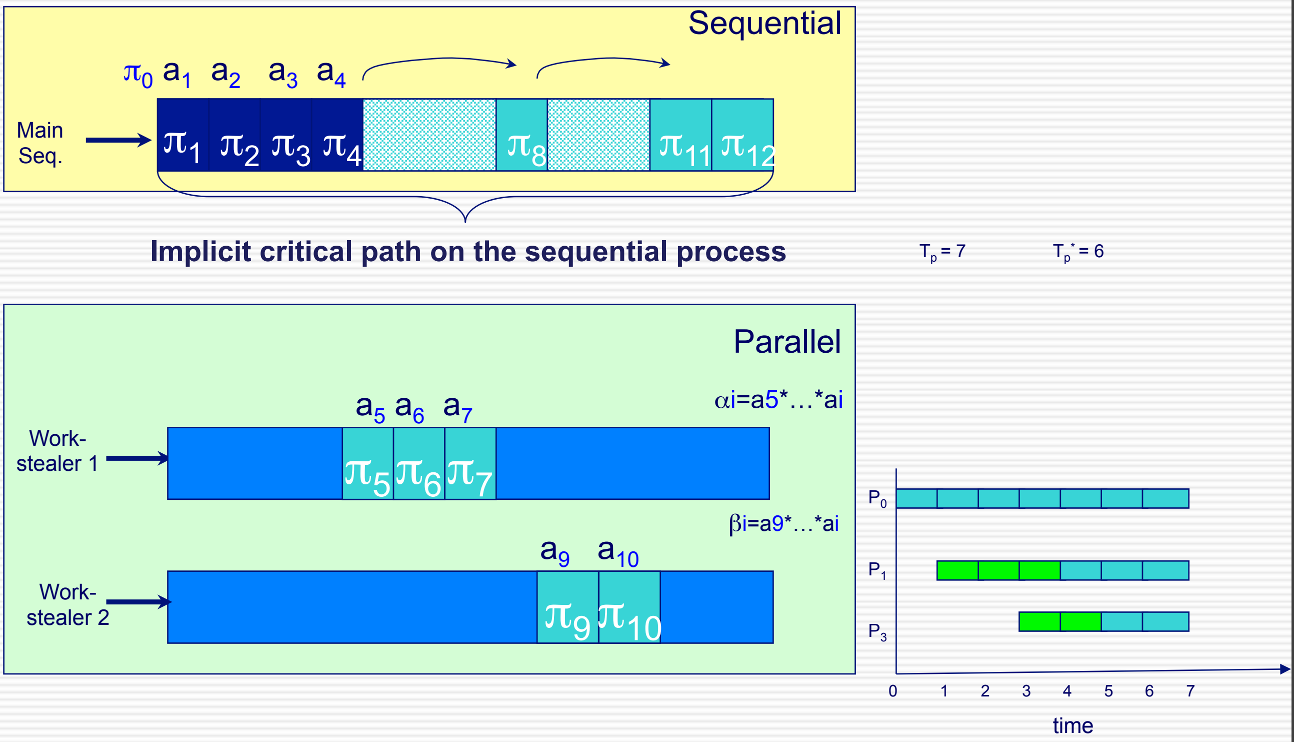


Analysis of the algorithm

- $$\text{Execution time} \leq \frac{2n}{(p+1).\Pi_{ave}} + O\left(\frac{\log n}{\Pi_{ave}}\right)$$

  Lower bound

- Sketch of the proof :

  Dynamic coupling of two algorithms that complete simultaneously:

  - Sequential: (optimal) number of operations S on one processor
  - Extract_par : work stealer perform X operations on other processors
    - dynamic splitting always possible till finest grain BUT local sequential
      - Critical path small (eg : log X with a W= n / log* n macroloop)
      - Each non constant time task can potentially be splitted (variable speeds)

  $$T_s = \frac{S}{\Pi_{ave}} \text{ and } T_p = \frac{X}{(p-1).\Pi_{ave}} + O\left(\frac{\log X}{\Pi_{ave}}\right)$$

  - Algorithmic scheme ensures $T_s = T_p + O(\log X)$

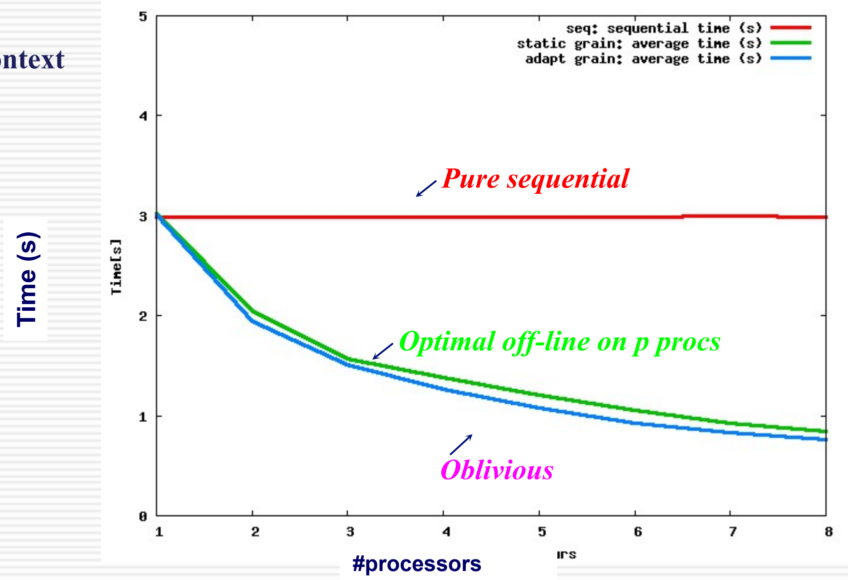
    => enables to bound the whole number X of operations performed
    and the overhead of parallelism = (s+X) - #ops_optimal

Results 1/2

[D Traore]

Prefix sum of 8.10^6 double on a SMP 8 procs (IA64 1.5GHz/ linux)

Single user context



Single-usercontext : processor-oblivious prefix achieves near-optimal performance :

- close to the lower bound both on 1 proc and on p processors

- Less sensitive to system overhead : even better than the theoretically "optimal" off-line parallel algorithm on p processors :

Results 2/2

[D Traore]

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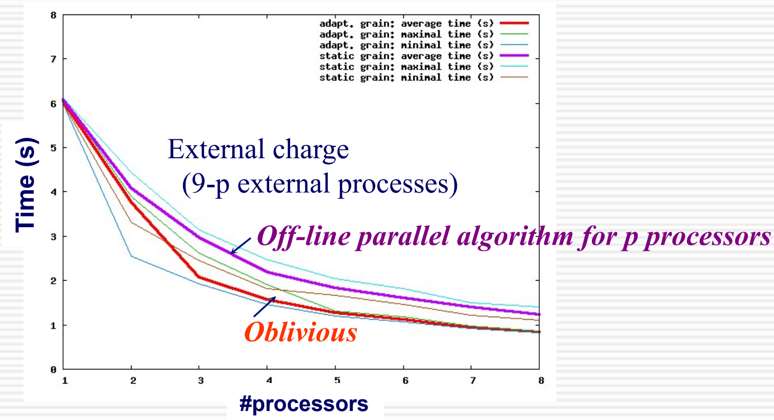
Multi-user context :



Multi-user context :
Additional external charge: (9-p) additional external dummy processes are concurrently executed

Processor-oblivious prefix computation is always the fastest
15% benefit over a parallel algorithm for p processors with off-line schedule,

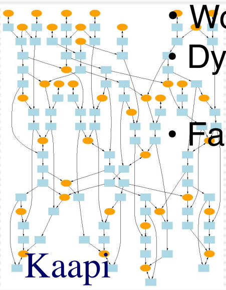
Conclusion

- **Fine grain parallelism enables efficient execution on a small number of processors**
  - Interest : portability ; mutualization of code ;
  - Drawback : needs work-first principle => algorithm design
- **Efficiency of classical work stealing relies on *work-first principle* :**
  - Implicitly defenerates a parallel algorithm into a sequential efficient ones ;
  - Assumes that parallel and sequential algorithms perform about the same amount of operations
- **Processor Oblivious algorithms based on *work-first principle***
  - Based on anytime extraction of parallelism from any sequential algorithm (may execute different amount of operations) ;
  - Oblivious: near-optimal whatever the execution context is.
- **Generic scheme for stream computations :**

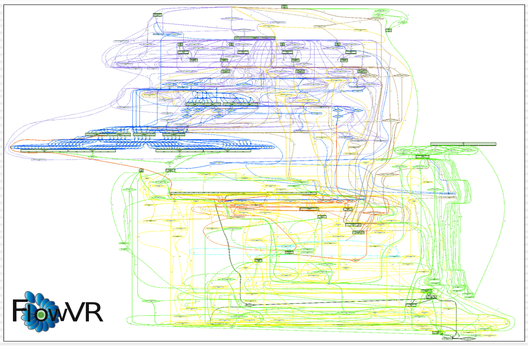
  parallelism introduce a copy overhead from local buffers to the output

  gzip / compression, MPEG-4 / H264

Kaapi (kaapi.gforge.inria.fr)



- Work stealing / work-first principle
- Dynamics Macro-dataflow :
  partitioning (Metis, …)
- Fault Tolerance (add/del resources)



FlowVR (flowvr.sf.net)

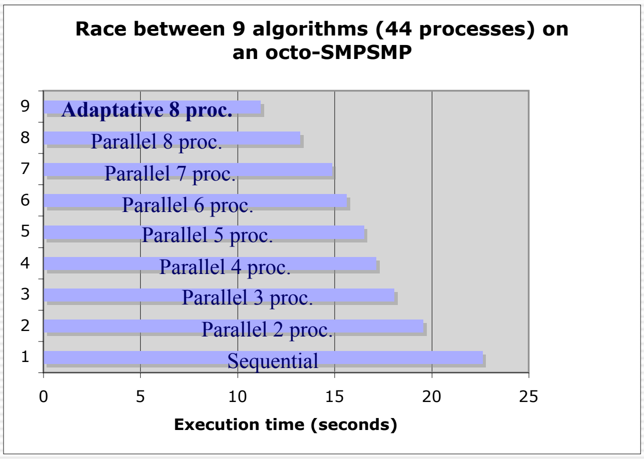
- Dedicated to interactive applications
- Static Macro-dataflow
- Parallel Code coupling

[E Boyer, B Raffin 2006]

Thank you !

Back slides

The Prefix race: sequential/parallel fixed/ adaptive

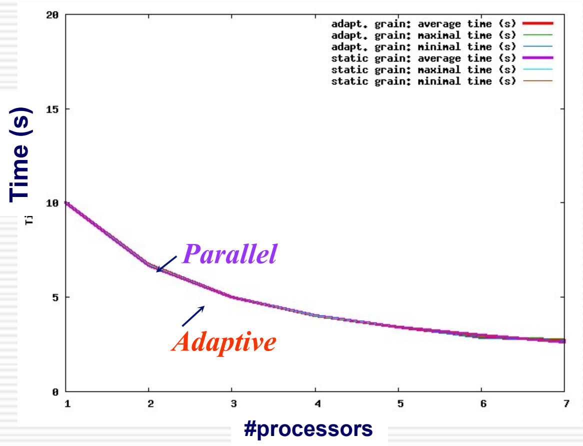


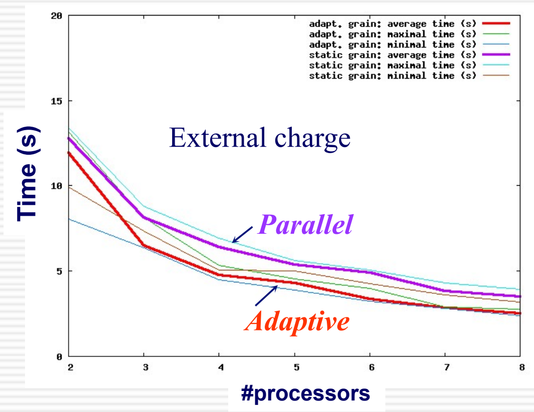
	Sequentiel	Statique					Adaptatif
		p=2	p=4	p=6	p=7	p=8	p=8
Minimum	21,83	18,16	15,89	14,99	13,92	12,51	8,76
Maximum	23,34	20,73	17,66	16,51	15,73	14,43	12,70
Moyenne	22,57	19,50	17,10	15,58	14,84	13,17	11,14
Mediane	22,58	19,64	17,38	15,57	14,63	13,11	11,01

On each of the 10 executions, adaptive completes first

Adaptive prefix : some experiments

Prefix of 10000 elements on a SMP 8 procs (IA64 / linux)





Single user context

Adaptive is equivalent to:

- sequential on 1 proc
- optimal parallel-2 proc. on 2 processors
- …
- optimal parallel-8 proc. on 8 processors

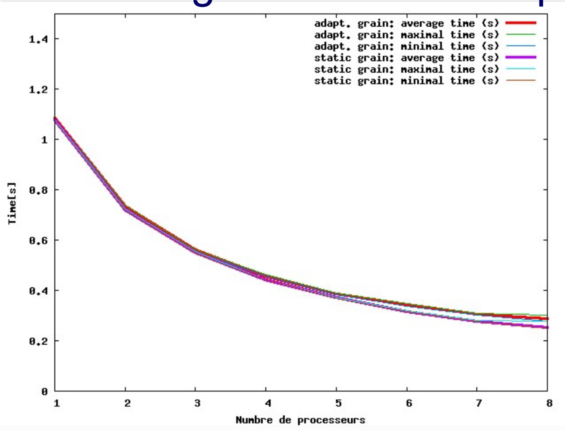
Multi-user context

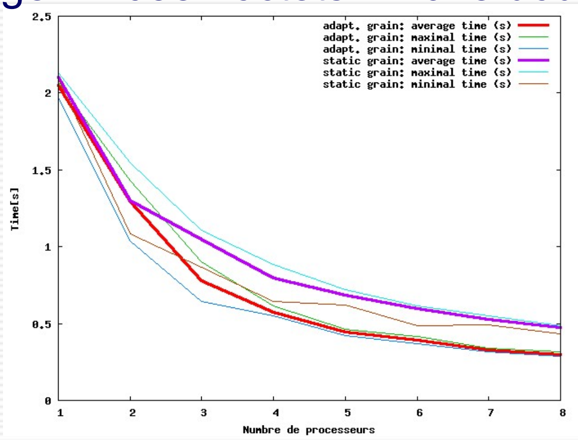
Adaptive is the fastest

15% benefit over a static grain algorithm

With * = double sum (r[i]=r[i-1] + x[i])

Finest "grain" limited to 1 page = 16384 octets = 2048 double





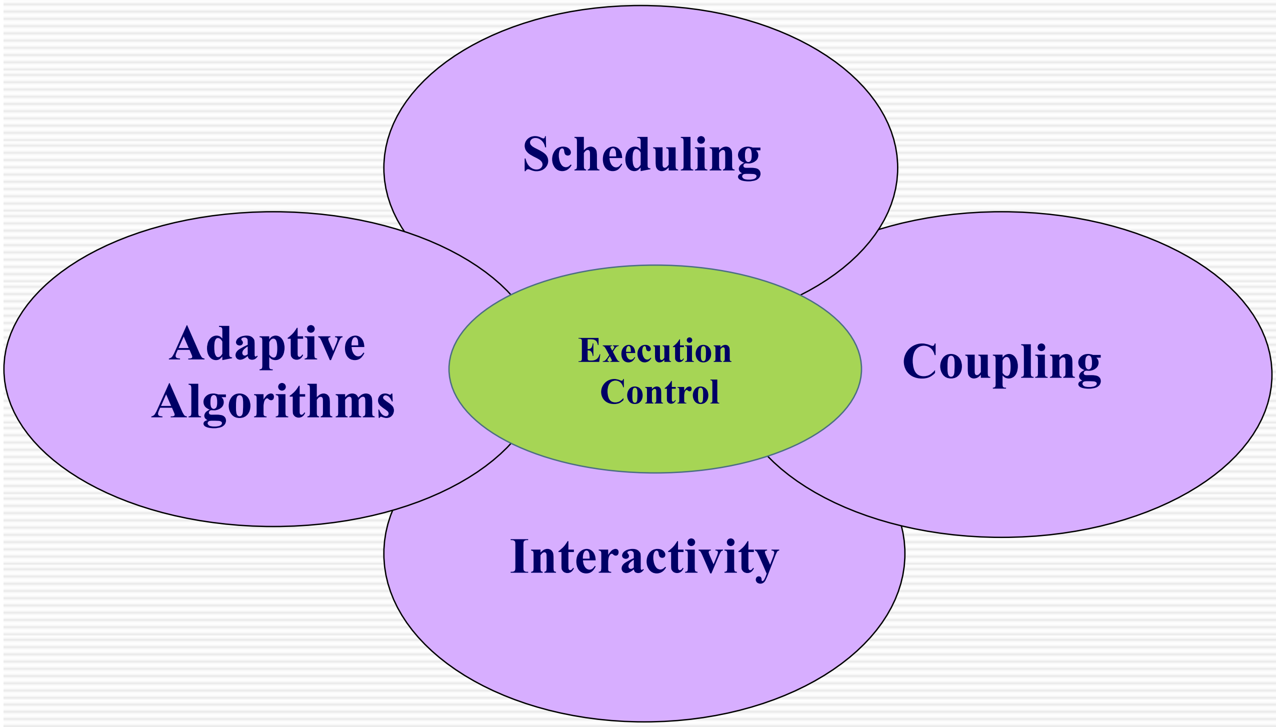
Single user

Processors with variable speeds

Remark for n=4.096.000 doubles :

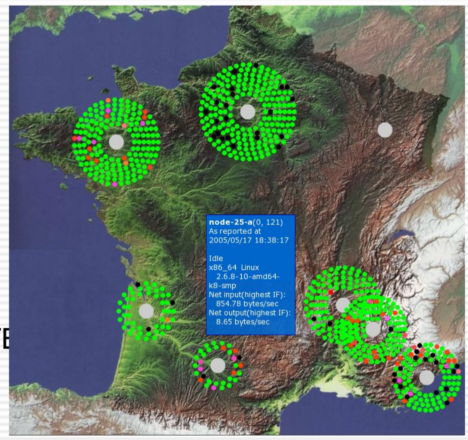
- "pure" sequential : 0,20 s
- minimal "grain" = 100 doubles : 0.26s on 1 proc
  and 0.175 on 2 procs (close to lower bound)

The Moais Group



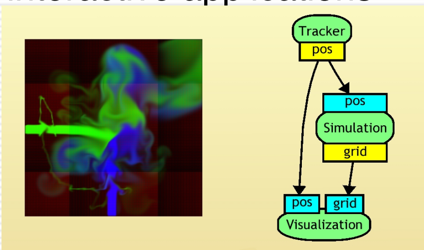
Moais Platforms

- Icluster 2 :
  - 110 dual Itanium bi-processors with Myrinet network
- GrImage (“Grappe” and Image):
  - Camera Network
  - 54 processors (dual processor cluster)
  - Dual gigabits network
  - 16 projectors display wall
- Grids:
  - Regional: Ciment
  - National: Grid5000
    - Dedicated to CS experiments
- SMPs:
  - 8-way Itanium (Bull novascale)
  - 8-way dual-core Opteron + 2 GPUs
- MPSoCs
  - Collaborations with ST Microelectronics on STE



Parallel Interactive App.

- Human in the loop
- Parallel machines (cluster) to enable large interactive applications
- Two main performance criteria:
  - Frequency (refresh rate)
    - Visualization: 30-60 Hz
    - Haptic : 1000 Hz
  - Latency (makespan for one iteration)
    - Object handling: 75 ms



- A classical programming approach: data-flow model
  - Application = static graph
    - Edges: FIFO connections for data transfert
    - Vertices: tasks consuming and producing data
    - Source vertices: sample input signal (cameras)
    - Sink vertices: output signal (projector)
- One challenge:
  Good mapping and scheduling of tasks on processors