Mapping filter services on heterogeneous platforms

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June 2, 2009

Introduction

The problem:

- processing of a data flow
- filter services with different selectivities and costs
- precedence constraints between services
- servers with different speeds
- one-to-one mappings

The objective:

- minimizing period
- minimizing latency
- bi-criteria: minimize latency for a fixed period

The model

Example: In a list L of numbers:

- a first filter removes odd numbers ($\sigma_1 = 1/2$)
- a second filter transmits only multiples of 3 ($\sigma_1 = 1/3$)

The resulting list contains all multiples of 6 in L.

The model

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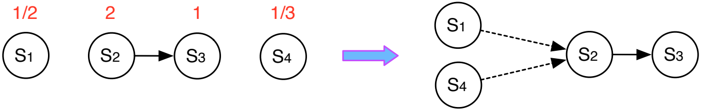
The resulting list contains all multiples of 6 in L.
Hypotheses:

- No communication cost
- Join operation of null cost
- Selectivities are independent

Playing with selectivities

- Service S_i transforms (filters) data of size δ to size $\sigma_i \times \delta$
- Computation cost depends on the data size (previous σ)
- May add dependencies to exploit selectivity



- S_1 and S_4 process file of initial size 1; S_1 removes even numbers and S_2 removes not multiples of 3.
- Combined file of size $\frac{1}{2}.\frac{1}{3} = \frac{1}{6}$ (no cost for join)
- S_2 duplicates the file
- S_3 processes but does not alter the file

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The instances

The problems depend on:

- the criteria: MinPeriod, MinLatency or BiCriteria
- the platform: Hom or Het
- the dependence constraints: NoPrec or Prec

The instances

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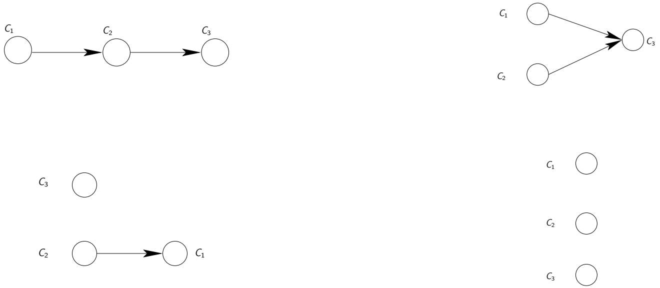
- the criteria: MinPeriod, MinLatency or BiCriteria
- the platform: Hom or Het
- the dependence constraints: NoPrec or Prec

The instances: $\mathcal{A} = (\mathcal{F}, \mathcal{G}, \mathcal{S})$ with:

- The services: $\mathcal{F} = \{C_1, C_2, \dots, C_n\}$
- The precedence constraints: $\mathcal{G} \subset \mathcal{F} \times \mathcal{F}$
- The servers: $\mathcal{S} = \{S_1, S_2, \dots, S_p\}$

The problem

Example for 3 independent services:
The plan?



The mapping?

$$(C_1, S_2), (C_2, S_1), (C_3, S_3)$$

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Example

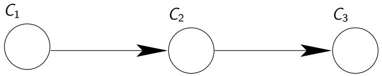

Figure: Chaining services.

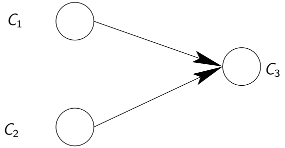

Figure: Combining selectivities

$$\mathcal{P} = \max\left(\frac{c_1}{s_1}, \frac{\sigma_1 c_2}{s_2}, \frac{\sigma_1\sigma_2 c_3}{s_3}\right) \qquad \mathcal{P} = \max\left(\frac{c_1}{s_1}, \frac{c_2}{s_2}, \frac{\sigma_1\sigma_2 c_3}{s_3}\right)$$

$$\mathcal{L} = \frac{c_1}{s_1} + \frac{\sigma_1 c_2}{s_2} + \frac{\sigma_1\sigma_2 c_3}{s_3} \qquad \mathcal{L} = \max\left(\frac{c_1}{s_1}, \frac{c_2}{s_2}\right) + \frac{\sigma_1\sigma_2 c_3}{s_3}$$

Example

- $c_1 = 1,\ c_2 = 4,\ c_3 = 10$
- $\sigma_1 = \frac{1}{2},\ \sigma_2 = \sigma_3 = \frac{1}{3}$
- $s_1 = 1,\ s_2 = 2$ and $s_3 = 3$

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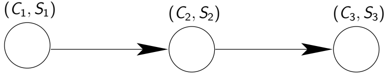

Figure: Optimal plan for period.

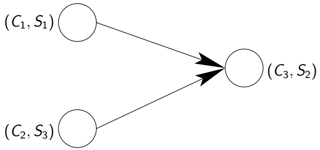

Figure: Optimal plan for latency

$$P = 1$$
$$L = \frac{5}{2}$$

$$L = \frac{13}{6}$$
$$P = \frac{4}{3}$$

General structure of optimal solutions

The instance : $C_1, ..., C_n, S_1, ..., S_n$ with

- $\sigma_1, ..., \sigma_p \leq 1$
- $\sigma_{p+1}, ..., \sigma_n \geq 1$

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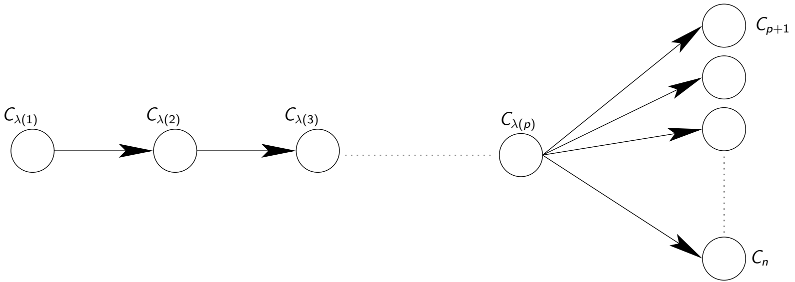

Figure: General structure

Homogeneous case without precedence constraints

The instance : $C_1, ..., C_n$ with

- $c_1 \leq c_2 \leq ... \leq c_p$
- $\sigma_1, ..., \sigma_p < 1$
- $\sigma_{p+1}, ..., \sigma_n \geq 1$

The matching: $C_1 \rightarrow C_2 \rightarrow ... \rightarrow C_p$

Homogeneous case with precedence constraints

Computing the optimal subgraph for C in the graph G is polynomial. Paper [Srivastava et al] presents a polynomial time algorithm using a min-cut algorithm.

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Paper [Srivastava et al] presents a polynomial time algorithm using a min-cut algorithm.
Optimal algorithm for period in homogeneous case:
We add the nodes step by step.
At each step, we place the available service with minimal possible period.

Proof: NP-completeness of MinPeriod-Het

Problem (RN3DM)

Given an integer vector $A = (A[1], \ldots, A[n])$ of size n, does there exist two permutations λ_1 and λ_2 of $\{1, 2, \ldots, n\}$ such that

$$\forall 1 \leq i \leq n, \quad \lambda_1(i) + \lambda_2(i) = A[i]$$

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The associated instance :

- $c_i = 2^{A[i]}$
- $\sigma_i = 1/2$
- $s_i = 2^i$
- $P = 2$

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$$\begin{aligned} & \forall 1 \leq i \leq n, \quad \lambda_1(i) + \lambda_2(i) \geq A[i] \\ \iff \; & \forall 1 \leq i \leq n, \quad \left(\frac{1}{2}\right)^{\lambda_1(i)-1} \times \frac{2^{A[i]}}{2^{\lambda_2(i)}} \leq 2 \end{aligned}$$

Inapproximability of MinPeriod-Het

Proposition

For any $K > 0$, there exists no K-approximation algorithm for MinPeriod-NoPrec-Het, *unless P=NP.*

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Reduction from RN3DM:

- $c_i = K^{A[i]-1}$
- $\sigma_i = 1/K$
- $s_i = K^i$
- $P = 1$

Integer linear program

The variables:

- $t_{i,u} = 1$ if service C_i is assigned to server S_u
- $s_{i,j} = 1$ if service C_i is an ancestor of C_j
- M is the logarithm of the optimal period

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- $t_{i,u} = 1$ if service C_i is assigned to server S_u
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- M is the logarithm of the optimal period

The constraints:

- $\forall i, \quad \sum_u t_{i,u} = 1$
- $\forall u, \quad \sum_i t_{i,u} = 1$
- $\forall i, j, k, \quad s_{i,j} + s_{j,k} - 1 \leq s_{i,k}$
- $\forall i, s_{i,i} = 0$
- $\forall i, \quad \log c_i - \sum_u t_{i,u} \log s_u + \sum_k s_{k,i} \log \sigma_k \leq M$

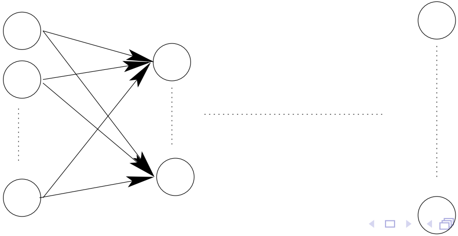
The objective function: Minimize M

Structure of the optimal plan

Proposition

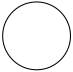
Let $C_1, \dots, C_n, S_1, \dots, S_n$ be an instance of MinLatency. *Then, the optimal latency is obtained with a plan G such that, for any $v_1 = (C_{i_1}, S_{u_1})$, $v_2 = (C_{i_2}, S_{u_2})$,*

1. *If $d_{i_1}(G) = d_{i_2}(G)$, they have the same predecessors and the same successors in G.*
2. *If $d_{i_1}(G) > d_{i_2}(G)$ and $\sigma_{i_2} \leq 1$, then $c_{i_1}/s_{u_1} < c_{i_2}/s_{u_2}$.*
3. *All nodes with a service of selectivity $\sigma_i > 1$ are leaves ($d_i(G) = 0$).*



Principle of the algorithm

C_1



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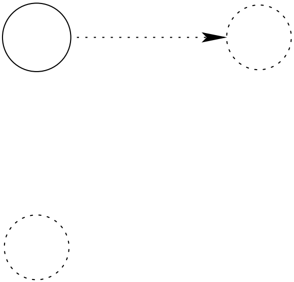

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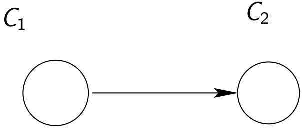

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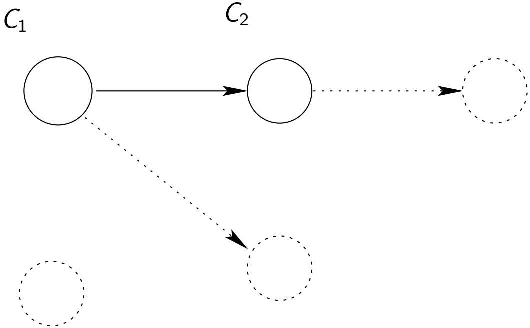

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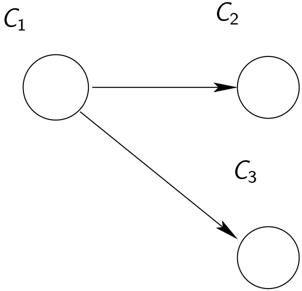

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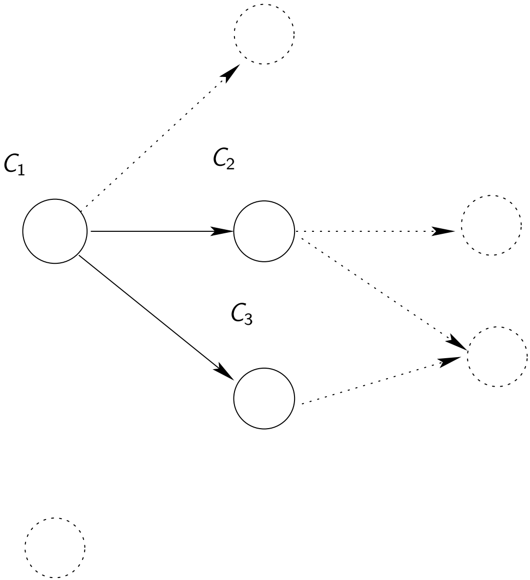

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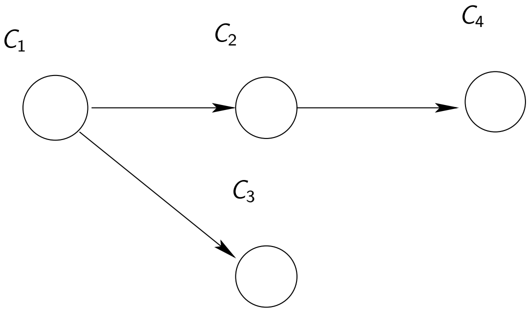

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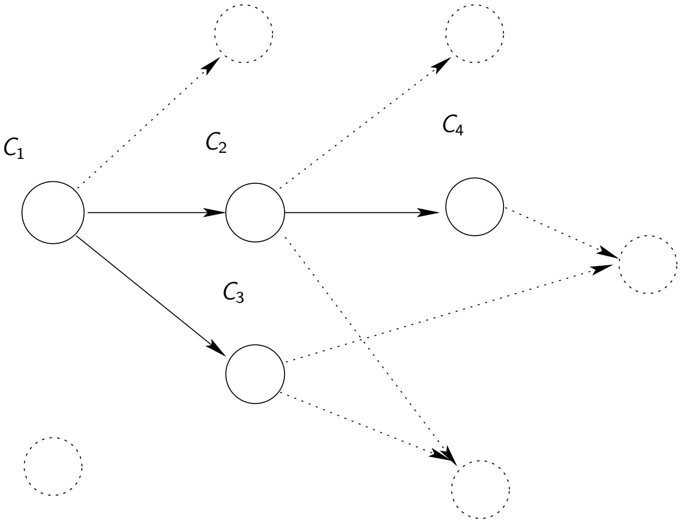

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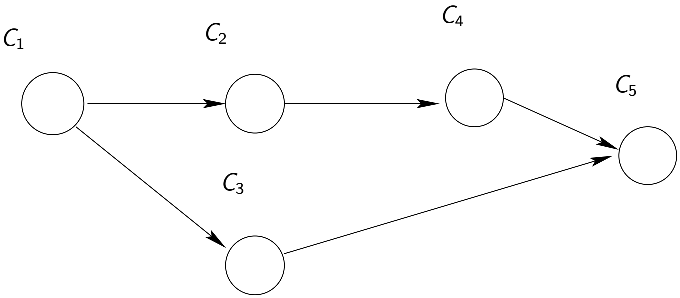

Algorithm without dependence constraints

Data: n services of costs $c_1 \leq \cdots \leq c_n$ and of selectivities $\sigma_1, ..., \sigma_n \leq 1$
Result: a plan G optimizing the latency
G is the graph reduced to node C_1;
for $i = 2$ **to** n **do**
    for $j = 0$ **to** $i - 1$ **do**
        Compute the completion time t_j of C_i in G with predecessors $C_1, ..., C_j$;
    end
    Choose j such that $t_j = \min_k\{t_k\}$;
    Add the node C_i and the edges $C_1 \rightarrow C_i, \ldots, C_j \rightarrow C_i$ to G;
end

Algorithm 1: Optimal algorithm for MinLatency-NoPrec-Hom.

G is the graph reduced to the node C of minimal cost with no predecessor in $\mathcal{G}$;

for $i = 2$ **to** n **do**

Let S be the set of services not yet in G and such that their set of predecessors in $\mathcal{G}$ is included in G;

for $C \in S$ **do**

for $C' \in G$ **do**

Compute the set S' minimizing the product of selectivities among services of latency less than $L_G(C')$, and including all predecessors of C in $\mathcal{G}$;

end

Let S_C be the set that minimizes the latency of C in G and L_C be this latency;

end

Choose a service C such that $L_C = \min\{L_{C'}, C' \in S\}$;

Add to G the node C, and $\forall C' \in S_C$, the edge $C' \rightarrow C$;

end

Algorithm 2: Optimal algorithm for MINLATENCY-PREC-HOM.

1. Framework
2. Period
   - General structure of optimal solutions
   - Case of homogeneous servers
   - NP-completeness of MinPeriod-Het
   - Integer linear program
3. Latency
   - General structure of optimal solutions
   - Polynomial algorithm on homogeneous platforms
   - NP-completeness of problem MinLatency-NoPrec-Het
   - Integer linear program
4. Bi-criteria problem
5. Conclusion

Proof:NP-completeness of MinLatency-Het

Lemma

Let $C_1, ..., C_n, S_1, ..., S_n$ be an instance such that $\forall i$, c_i and s_i are integer power of 2 and $\sigma_i \leq \frac{1}{2}$. Then the optimal latency is obtained with a plan G such that

1. *Proposition 2 is verified;*
2. *for all nodes (C_{i_1}, S_{u_1}) and (C_{i_2}, S_{u_2}) with $d_{i_1}(G) = d_{i_2}(G)$, we have $\frac{c_{i_1}}{s_{u_1}} = \frac{c_{i_2}}{s_{u_2}}$.*

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- $c_i = 2^{A[i] \times n + (i-1)}$
- $\sigma_i = \left(\frac{1}{2}\right)^n$
- $s_i = 2^{n \times (i+1)}$
- $L = 2^n - 1$

Inapproximability of MinLatency-Het

Proposition

For any $K > 0$, there exists no K-approximation algorithm for MinLatency-NoPrec-Het, *unless P=NP.*

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Reduction from RN3DM

- $c_i = K^{A[i] \times n + (i-1)}$
- $\sigma_i = \left(\frac{1}{K}\right)^n$
- $s_i = K^{n \times (i+1)}$
- $L = \frac{K^n - 1}{K - 1}$

The variables:

- $z(i, u, e) = 1$ if the service C_i is associated to the server S_u and its set of predecessors is $e \subset \mathcal{C}$.
- $t(i)$ is the completion time of C_i
- M is the optimal latency

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- $z(i, u, e) = 1$ if the service C_i is associated to the server S_u and its set of predecessors is $e \subset \mathcal{C}$.
- $t(i)$ is the completion time of C_i
- M is the optimal latency

The constraints:

- $\forall u \in \mathcal{S}, \quad \sum_{i \in \mathcal{C}} \sum_{e \subset \mathcal{C}} z(i, u, e) = 1$
- $\forall i \in \mathcal{C}, \quad \sum_{u \in \mathcal{S}} \sum_{e \subset \mathcal{C}} z(i, u, e) = 1$
- $\forall i, i' \in \mathcal{C}, \forall u, u' \in \mathcal{S}, \forall e, e' \subset \mathcal{C}, e \not\subseteq e', i \in e', \quad z(i, u, e) + z(i', u', e') \leq 1$
- $\forall u \in \mathcal{S}, \forall e \subset \mathcal{C}, \forall i \in e, \ z(i, u, e) = 0$
- $\forall i \in \mathcal{C}, \forall e \subset \mathcal{C}, \forall k \in e, \quad t(i) \geq \sum_{u \in \mathcal{S}} z(i, u, e) \left(\frac{c_i}{s_u} * \prod_{C_j \in e} \sigma_j + t(k) \right)$
- $\forall i \in \mathcal{C}, \quad t(i) \geq \sum_u z(i, u, e) \frac{c_i}{s_u} * \prod_{C_j \in e} \sigma_j$
- $\forall i \in \mathcal{C}, \quad t(i) \leq M$

The objective function: Minimize M

Data: n services of costs $c_1 \leq \cdots \leq c_n$ and of selectivities $\sigma_1, ..., \sigma_n \leq 1$ and a maximum throughput K

Result: a plan G optimizing the latency with a throughput less than K

G is the graph reduced to node C_1;

for $i = 2$ **to** n **do**

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  end

  Let $S = \{k | c_i \prod_{0 \leq k < i} \sigma_k \leq K\}$;

  Choose j such that $t_j = \min_{k \in S}\{t_k\}$;

  Add the node c_i and the edges $C_1 \rightarrow C_i, \ldots, C_j \rightarrow C_i$ to G;

end

Algorithm 3: Optimal algorithm for latency with a fixed throughput.

Conclusion

The results:

- MinLatency-Hom is polynomial
- MinPeriod-Het is NP-hard
- MinLatency-Het is NP-hard
- BiCriteria-Hom is polynomial

Future work:

- Model with communication costs (see SPAA'09)