

Mapping of the FDTD computations in a streaming model architecture

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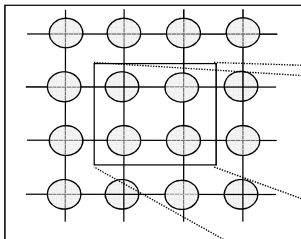
Outline

1. Introduction to the FDTD (*Finite Difference Time Domain*) problem
2. Loop tiling theory - a tile, a dependency set, a space iteration graph
3. Loop tiling in the FDTD problem
4. Streaming computational model
5. Example of streaming architecture – PS3 Cell/BE processor
6. Experimental results
7. Conclusions

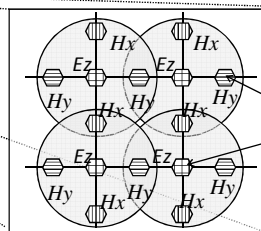
Introduction

- The Finite Difference Time Domain (FDTD) method is an iterative algorithm, which enables performing a simulation of the electromagnetic wave propagation.

Mesh of physical points



Computational mesh



Computational cells

- It is done by solving Maxwell equations.

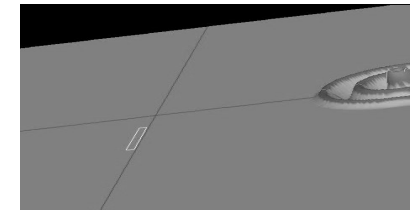
$$\nabla \times H = \gamma E + \epsilon \frac{\partial E}{\partial t}, \quad \nabla \times E = -\mu \frac{\partial H}{\partial t} \quad (1)$$

$$\overline{H}_x^i(i, j) = \overline{H}_x^{i-1}(i, j) + RC \cdot [\overline{E}_z^{n-0.5}(i, j-1) - \overline{E}_z^{n-0.5}(i, j+1)],$$

$$\overline{H}_y^i(i, j) = \overline{H}_y^{i-1}(i, j) + RC \cdot [\overline{E}_z^{n-0.5}(i-1, j) - \overline{E}_z^{n-0.5}(i+1, j)],$$

$$\overline{E}_z^i(i, j) = CA(i, j) \cdot \overline{E}_z^{i-1}(i, j) + CB(i, j) \cdot [\overline{H}_x^{i-0.5}(i+1, j) - \overline{H}_x^{i-0.5}(i-1, j) + \overline{H}_y^{i-0.5}(i, j-1) - \overline{H}_y^{i-0.5}(i, j+1)]$$

Introduction cnd.



$$\nabla \times H = \gamma E + \epsilon \frac{\partial E}{\partial t}, \quad \nabla \times E = -\mu \frac{\partial H}{\partial t} \quad (1)$$

$$\overline{H}_x^i(i, j) = \overline{H}_x^{i-1}(i, j) + RC \cdot [\overline{E}_z^{n-0.5}(i, j-1) - \overline{E}_z^{n-0.5}(i, j+1)],$$

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$$\overline{E}_z^i(i, j) = CA(i, j) \cdot \overline{E}_z^{i-1}(i, j) + CB(i, j) \cdot [\overline{H}_x^{i-0.5}(i+1, j) - \overline{H}_x^{i-0.5}(i-1, j) + \overline{H}_y^{i-0.5}(i, j-1) - \overline{H}_y^{i-0.5}(i, j+1)]$$



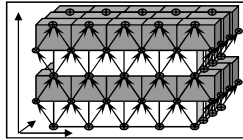
```

For (t=0; t<numberOfIterations; t++) {
  For (i=0; i<getMaxX(); i+=2)
    For (j=0; j<getMaxY(); j+=2)
      T[i][j]=C*T[i][j]+D*(T[i+1][j]-T[i-1][j]-T[i][j+1]-T[i][j-1])

  For (i=0; i<getMaxX(); i+=2)
    For (j=1; j<getMaxY()-1; j+=2)
      T[i][j]=T[i][j]+F*(T[i][j-1]-T[i][j+1])

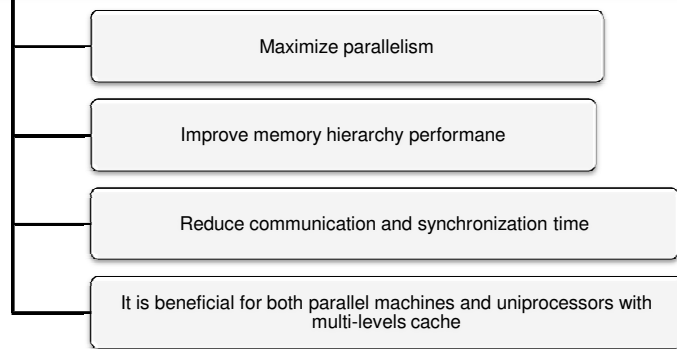
  For (i=1; i<getMaxX()-1; i+=2)
    For (j=1; j<getMaxY()-1; j+=2)
      T[i][j]=T[i][j]+F*(T[i-1][j]-T[i+1][j])
}
    
```

Loop tiling theory

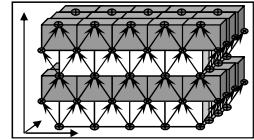


In „Loop tiling for parallelism” (Jingling Hue):

Loop tiling is one of the most important iteration-reordering loop transformation.



Loop tiling theory



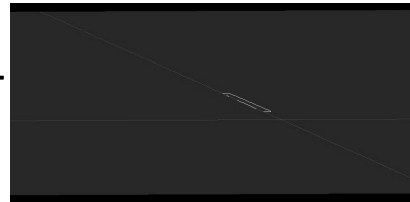
Example: Matrix-vector multiplication

```
for (i=0; i<N; i++)
    for (j=0; j<N; j++)
        c[i] = c[i] + a[i][j]*b[j];
```

Idea: Loop tiling partitions a loop's iteration space into smaller chunks or blocks

```
for (i=0; i<N; i+=3)
    for (j=0; j<N; j+=3)
        for (ii=i; ii<min(i+3,N); ii++)
            for (jj=j; jj<min(j+3,N); jj++)
                c[ii] =c[ii]+ a[ii][jj]*b[jj];
```

Loop tiling theory – Dependency set



```
For(t=0; t<numberOfIterations; t++) {
    For(i=0; i<getMaxX(); i+=2)
        For(j=0; j<getMaxY(); j+=2)
            T[i][j]=C*T[i][j]+D*(T[i+1][j]-T[i-1][j]-T[i][j+1]-T[i][j-1])

    For(i=0; i<getMaxX(); i+=2)
        For(j=1; j<getMaxY()-1; j+=2)
            T[i][j]=T[i][j]+F*(T[i][j-1]-T[i][j+1])

    For(i=1; i<getMaxX(); i+=2)
        For(j=1; j<getMaxY(); j+=2)
            T[i][j]=T[i][j]+F*(T[i-1][j]-T[i+1][j])
}
```

- $D_{ez} = \{(1, 0, 0), (1, -1, 0), (1, 1, 0), (1, 0, -1), (1, 0, 1)\}$
- $D_{hy} = \{(1, 0, 0), (1, 0, 1), (1, 0, -1)\}$
- $D_{hx} = \{(1, 0, 0), (1, 1, 0), (1, -1, 0)\}$

Dependency set for the two dimensional FDTD problem

FDTD tiling theory – iteration space graph

Dependency set

- $D_{ez} = \{(1, 0, 0), (1, -1, 0), (1, 1, 0), (1, 0, -1), (1, 0, 1)\}$
- $D_{hy} = \{(1, 0, 0), (1, 0, 1), (1, 0, -1)\}$
- $D_{hx} = \{(1, 0, 0), (1, 1, 0), (1, -1, 0)\}$

Distance Vectors
(Dependency vectors)

Direction value

$$D_{ez} = \{ \dots, \dots, \dots, (1, 0, -1), \dots \}$$

Time distance value

```
For(t=0; t<numberOfIterations; t++) {
    .....
```

Space distance values

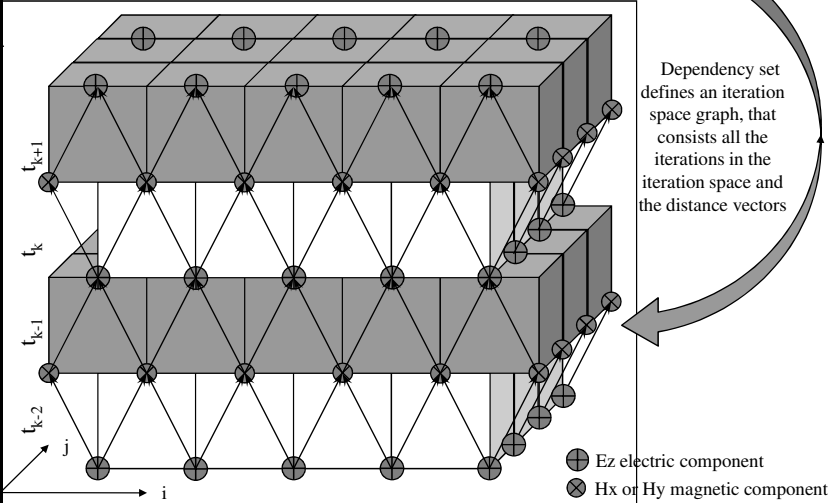
```
For(i=1; i<getMaxX(); i+=2) {
    For(j=1; j<getMaxY(); j+=2) {
        .....
```

FDTD tiling theory – iteration space graph

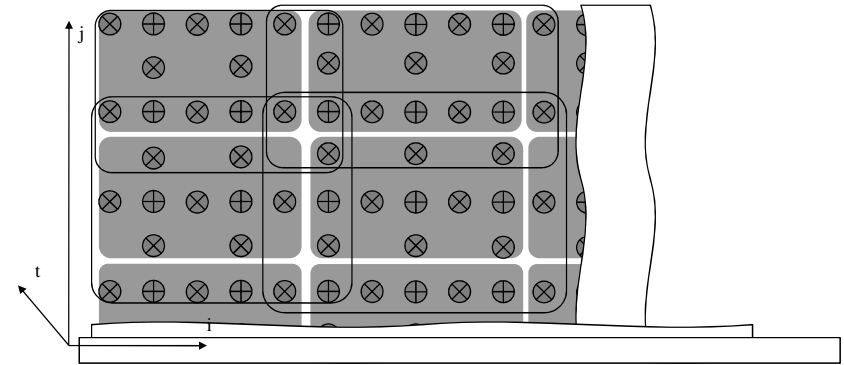
$$D_{ez} = \{(1, 0, 0), (1, -1, 0), (1, 1, 0), (1, 0, -1), (1, 0, 1)\}$$

$$D_{hy} = \{(1, 0, 0), (1, 0, 1), (1, 0, -1)\}$$

$$D_{hx} = \{(1, 0, 0), (1, 1, 0), (1, -1, 0)\}$$



FDTD tiling theory – loop space tiling



5x3 rectangular tiling of the FDTD ISG.

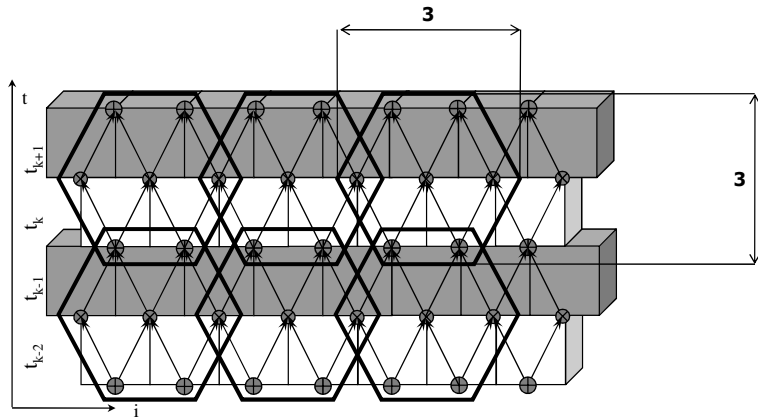
$$D_{ez} = \{(1, 0, 0), (1, -1, 0), (1, 1, 0), (1, 0, -1), (1, 0, 1)\}$$

$$D_{hy} = \{(1, 0, 0), (1, 0, 1), (1, 0, -1)\}$$

$$D_{hx} = \{(1, 0, 0), (1, 1, 0), (1, -1, 0)\}$$

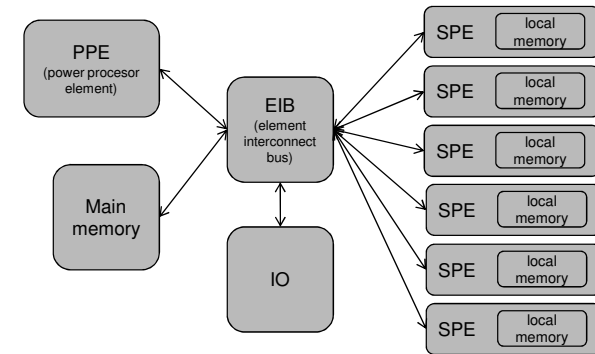
- Rectangular tile 5x3
- Overlapping boundary

FDTD tiling theory – loop time tiling

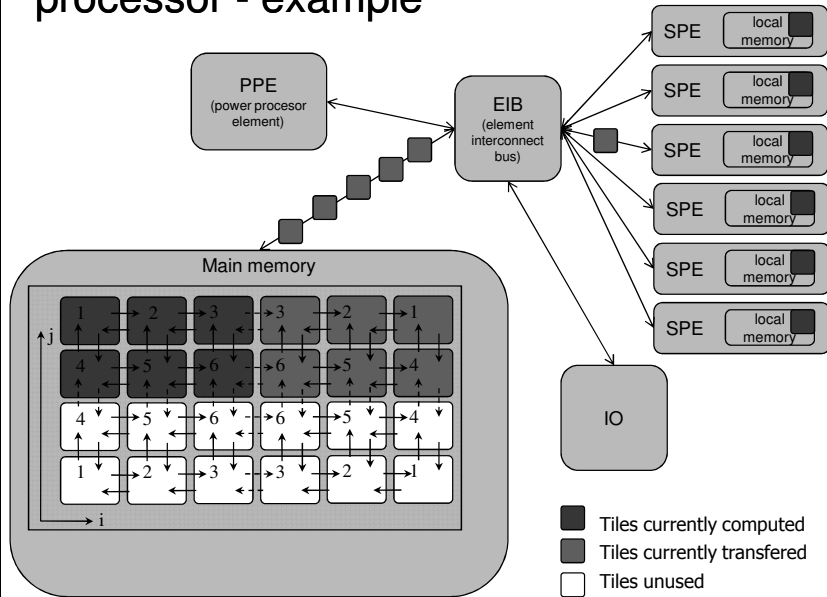


Hexagon shape time tiling for the FDTD 2D (height=3, width=3).

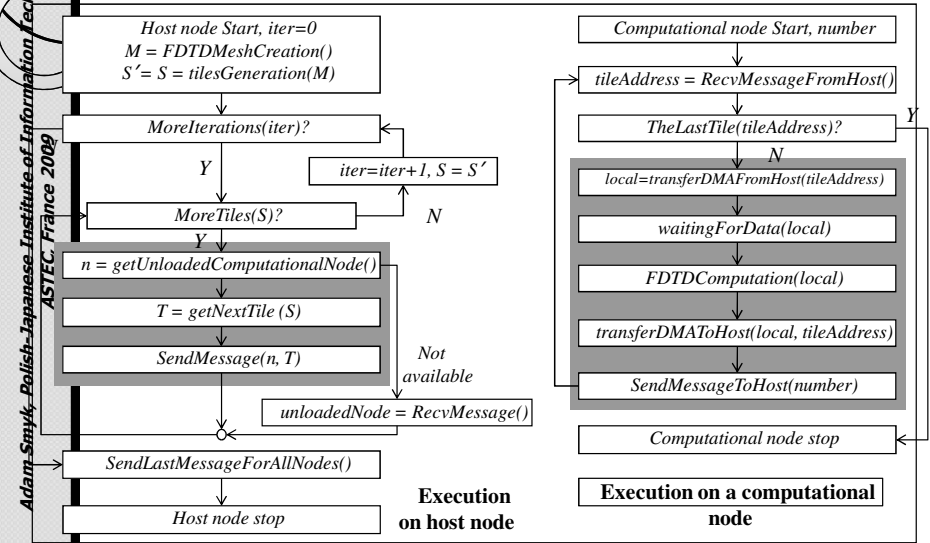
Streaming model architecture Cell/BE processor - example



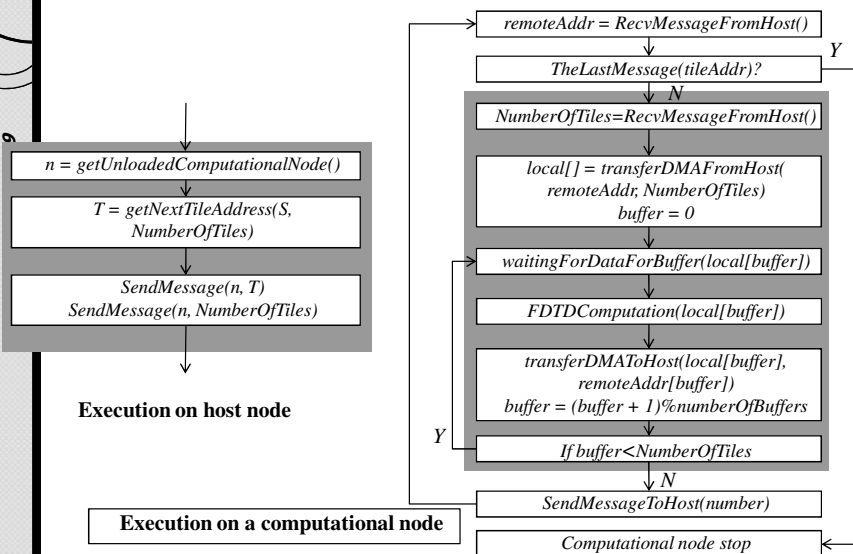
Streaming model architecture Cell/BE processor - example



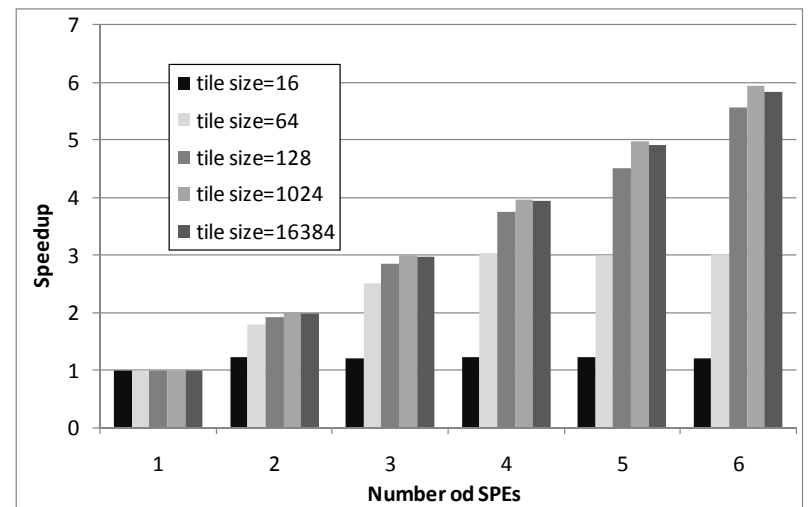
The FDTD computation scheme.



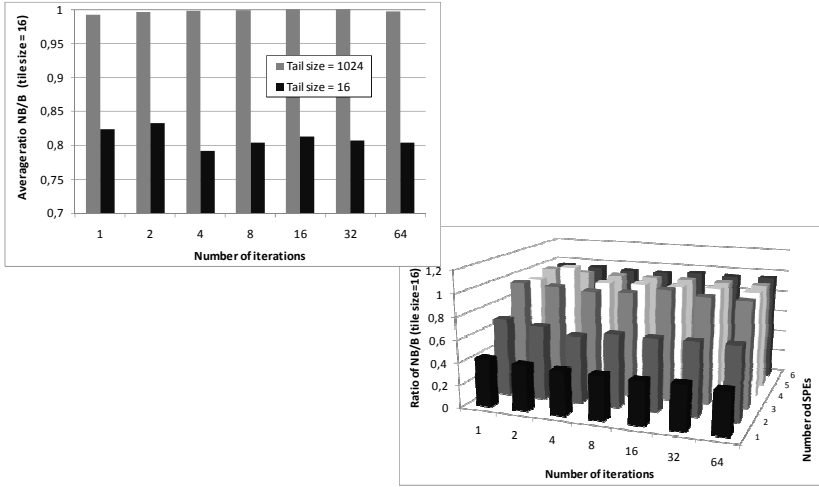
The FDTD computation scheme with rotating buffers modification.



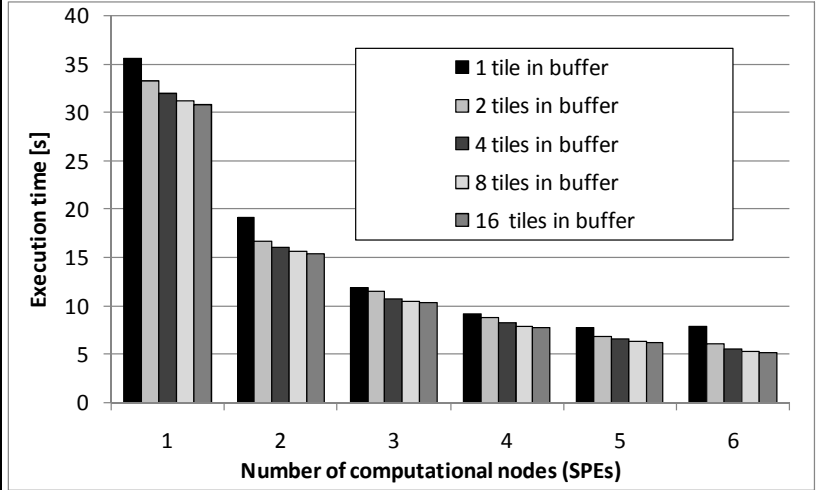
Speedup for FDTD computations for various tile sizes (space rectangular tiling).



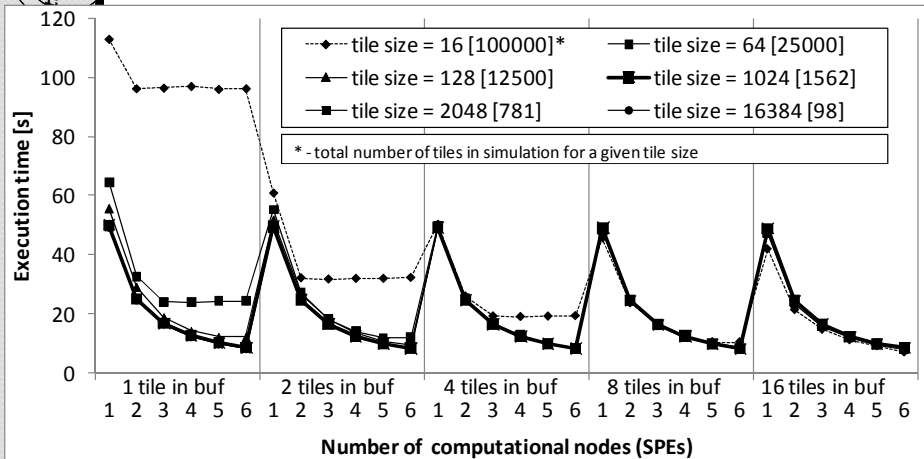
Comparison of efficiency of the FDTD computations with non-blocking and blocking mailbox communication.



Execution time of the FDTD computations with the rotating-buffers optimization (for tile size = 128)



Execution time for FDTD computation for various "tile" configurations.



Conclusions

- In the paper, the method of the FDTD simulations has been presented.
- Our idea is combining three optimization methods:
 - loop tiling theory,
 - streaming model computations
 - rotating buffers mechanism.
- We can consider a given problem from a programming structures point of view and from an architectural point of view:
 - The loop tiling method defines atomic portions of data spreading among all available computational nodes.
 - The rotating buffers mechanism has been used to create a pipeline of computations and communication, which provides a constant stream of data for computational nodes
- All these methods have been tested on the Cell/BE processor and they have given very promising results.
- Our experiments show that fine grain computations can be efficiently executed the Cell/BE processors using the streaming computational model.

Thank you !!!