Register allocation: What does the NP-completeness of Chaitin et al. really prove?

or

Revisiting register allocation: Why and how?

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Outline

1. NP-completeness of register allocation and ambiguities
   - Classical register allocation views
   - Example: iterated register coalescing
   - Confusions and questions
2. Determining if k registers are enough
   - NP-completeness proof of Chaitin et al.
   - Easy case: no critical edge, strict program, swaps
   - Where did the NP-completeness “disappear”?
3. Coalescing problems
   - Main results
   - Example: proof for optimistic coalescing
4. Conclusion

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What is register allocation?

Assign variables of a program to physical registers

- for a fixed instruction schedule;
- unlimited number of variables to place in:
  - a pool of limited resources (registers).
  - a pool of unlimited resources (memory).
- some architectural subtleties:
  - specific registers (e.g., sp, fp, r0);
  - variable affinities (e.g., auto-inc), register pairing (e.g., 64 bits operations);
  - distributed register banks, variable sizes, etc.
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Play with colors (registers) and color changes (transfers) with

register-to-register moves $\longrightarrow$ coalescing, live-range splitting;

insertions of stores and loads $\longrightarrow$ spilling.

What do we learn at school?

"Register allocation is NP-complete **because** graph coloring is NP-complete."

- variable ⇔ vertex;
- interferences between variables ⇔ edge;
- variable assignment ⇔ graph coloring.

⊳ use heuristics for register assignment (coloring), spilling (load/store insertion), and coalescing (move removal);

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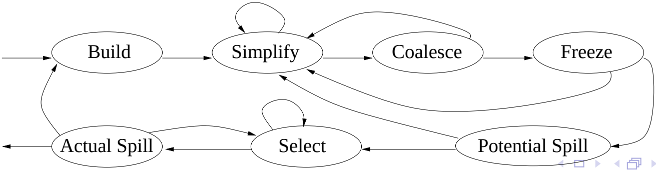
Hum. . . All this is confusing and misleading. We'll see why later.

Global register allocation with graph coloring:

Chaitin et al. (1981), Briggs-Cooper-Torczon (1994), Appel-George (2001), . . .

Given: k registers, interference graph, affinities (for coalescing).

- **Simplify** remove a non-move-related vertex with degree $< k$;
- **Coalesce** merge 2 move-related vertices (e.g., conservatively);
- **Freeze** give up about some moves;
- **Potential spill** remove a vertex and push it on a stack;
- **Select** pop a vertex and assign a color;
- **Actual spill** if no color is found, really insert load/store.



See power-point slides.

So, what is confusing?

Local register allocation is polynomial?

- Yes for deciding if k registers are enough, by renaming variables to get unique definitions.
- But what if more registers are needed, i.e., if some spilling is necessary?

  ☛ See Liberatore (1999) & Bouchez et al. (2005).

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- What about spilling? Coalescing?
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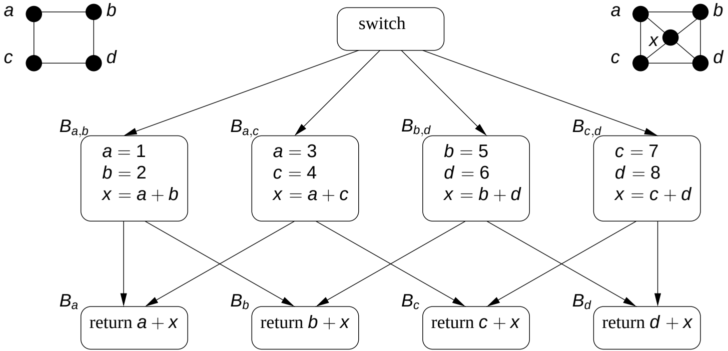
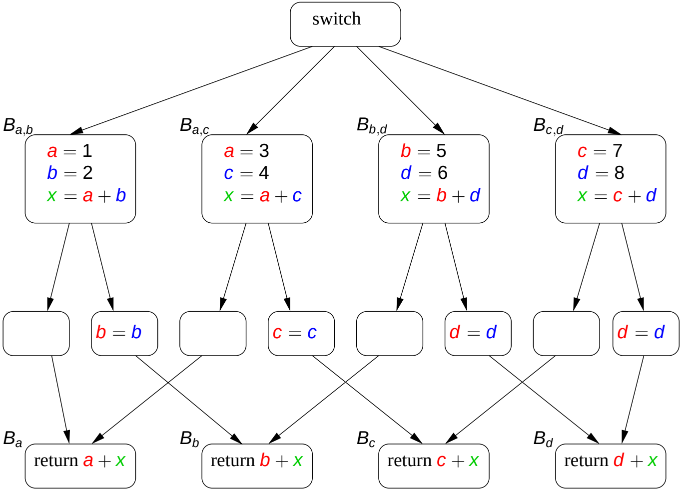
Interpretation of original proof



Chaitin et al. ⩾ NP-complete if each variable is assigned to a unique register.

Extension ⩾ if live-range splitting is allowed, remains NP-complete because of critical edges.

But proves nothing if blocks & moves can be inserted!



Maxlive: max. number of variables simultaneously live

Assume swaps, a strict program, edge splitting allowed

1. One needs Maxlive $\leq k$, so spill to get Maxlive $\leq k$.
2. Split critical edges (= add basic blocks).
3. Color each program point independently with at most Maxlive colors.
4. Use permutations to match colors (thanks to swaps).

This gives a correct assignment...

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More promising approaches

- Basic block coloring (interval graph);
- SSA-like coloring = subtrees of a tree (chordal graph);
- Guided live-range/edge splitting + permutation motion.

Pereira & Palsberg question (FOSSACS 2006)

After results by Brisk et al., Hack et al., Bouchez et al. on SSA and register allocation, Pereira and Palsberg wondered:

> " Can we do polynomial-time register allocation by first transforming the program to SSA form, then doing linear-time register allocation for the SSA form, and finally doing SSA elimination while maintaining the mapping from temporaries to registers? "

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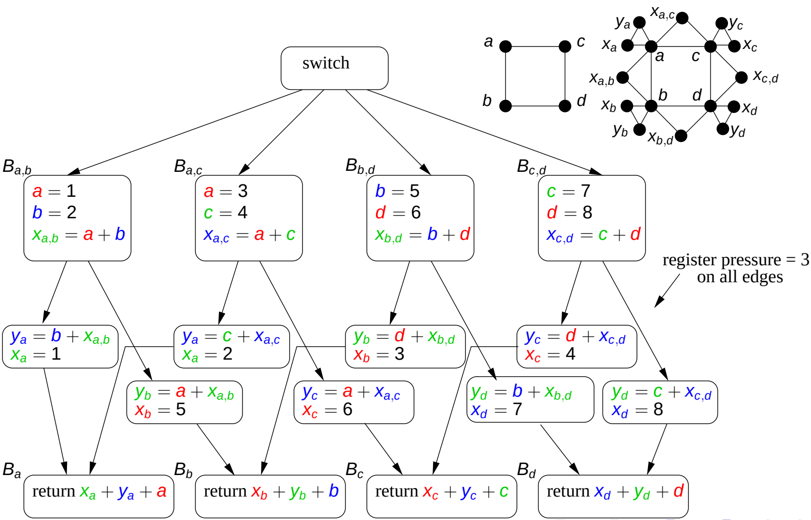
☛ They show it is NP-complete if swaps are not available.

- Reduce from k-coloring circular-arc graph.
- Make sure Live $= k$ on the back edge (where SSA will split) so that a non-trivial permutation is impossible.

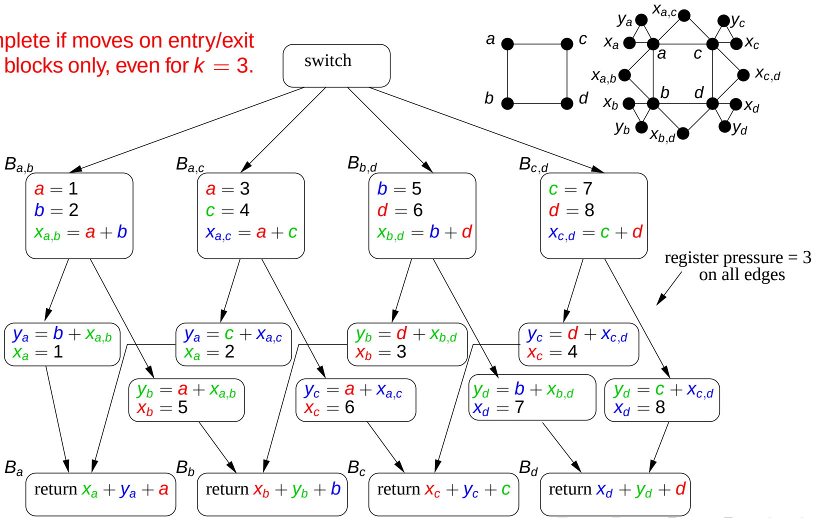
Note: polynomial for a fixed k. (See Garey, Johnson, Miller, Papadimitriou.)

Chaitin et al’s variant if swaps are not available



Chaitin et al’s variant if swaps are not available

NP-complete if moves on entry/exit of basic blocks only, even for $k = 3$.



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Replace each pair of definitions such as $y_a = b + x_{a,b}$ and $x_a = 1$ by one instruction $(x_a, y_a) = f(b, x_{a,b})$.

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So, NP-completeness did not disappear, it was simply not there! The proof of Chaitin et al. does not say anything about register allocation with live-range splitting and critical edge splitting.

On the complexity of register allocation

Register allocation remains difficult

- When critical edges cannot be split. But. . .
- Because optimal spilling is (almost) always hard.
- Because optimal coalescing is, in most cases, NP-complete.

☛ But, if moves are more suitable than loads and stores, it is in general easy to decide if some spilling is necessary or not.

Spill test Chaitin (degree $\geq k$) ➔ Briggs (potential spill) ➔ Appel-George (iterated) ➔ Biased coloring ➔ Optimal test

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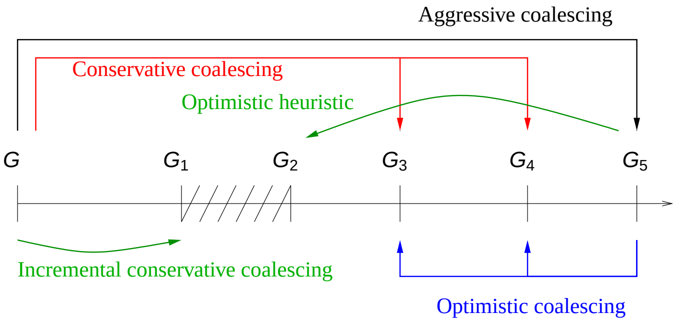
Links between different approaches



G: Initial graph, k-colorable or greedy-k-colorable
G_1: obtained by incremental conservative coalescing, greedy-k-colorable
G_2: obtained by optimistic de-coalescing, greedy-k-colorable
G_3: optimally coalesced greedy-k-colorable
G_4: optimally coalesced k-colorable
G_5: obtained by aggressive coalescing

Main complexity results

G interference graph, G_f graph after coalescing.

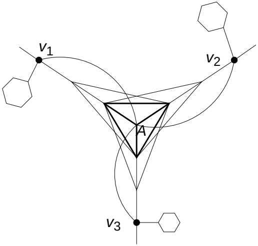
Aggressive coalescing NP-complete, even if G is chordal or greedy-3-colorable.

Conservative coalescing NP-complete even if G is greedy-2-colorable, one requires G_f to be chordal or greedy-3-colorable, and only affinities can be merged.

Incremental conservative coalescing (Briggs, George) NP-complete if G is arbitrary. Polynomial if G is chordal. Open if G is greedy-k-colorable.

Optimistic coalescing (Park & Moon) = conservative de-coalescing NP-complete even if G is chordal and $k = 4$.

Optimistic coalescing: from vertex cover, degree ≤ 3



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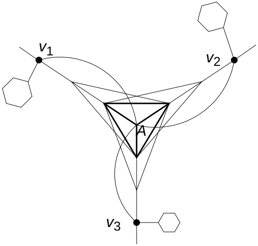

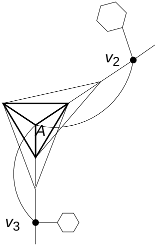

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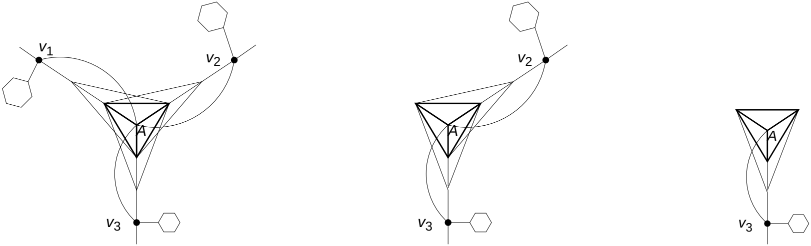

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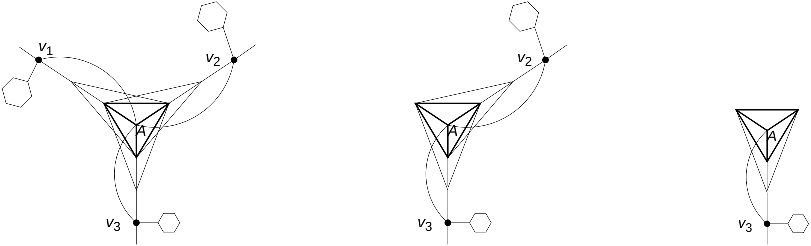
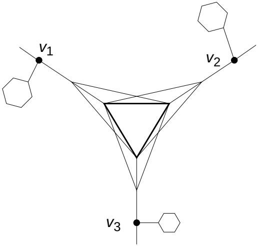
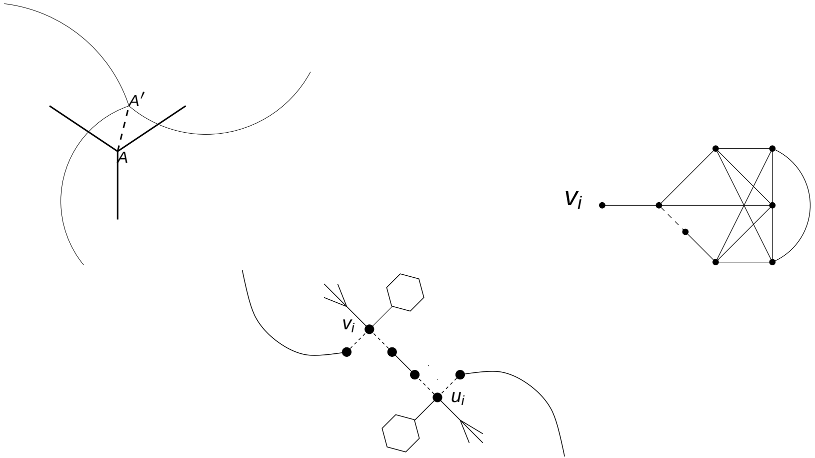


Remove central node →



Reduction for optimistic coalescing (Cont'd)



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Conclusions and future works

Be careful Chaitin et al. reduction from graph k-coloring does not really mean that “coloring” variables is hard.

Maxlive $\leq k$ is in general a good test for deciding if spilling is necessary. Even iterated register coalescing overspills.

Spilling is hard what to spill and where is challenging.

Splitting (some) critical edges does not seem to be a problem.

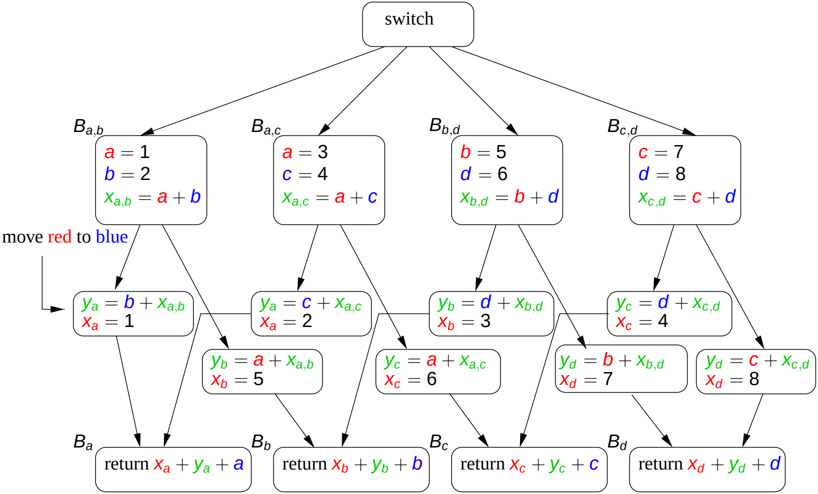
Spilling under SSA does not seem to be a good strategy.

Coalescing is hard in theory, even with “nice” graph structures. But good optimistic heuristics should be possible.

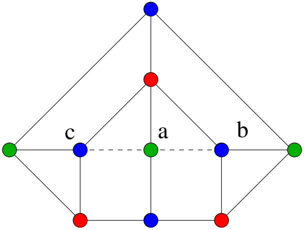
More experiments need to be done for exploring this new view and tradeoffs between spilling & coalescing.

That's all!
Any questions?

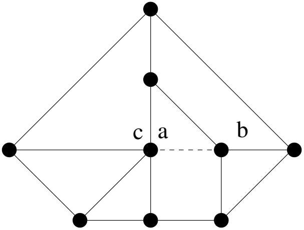
If moves can be anywhere, the proof is broken.



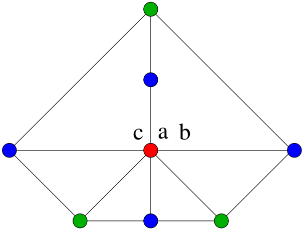
Failure of incremental conservative coalescing



greedy-3-colorable



greedy-4-colorable



greedy-3-colorable