Synthetic Load Injection

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Outline

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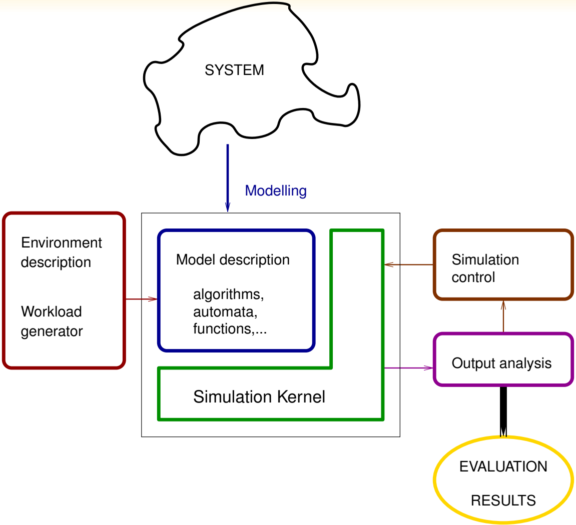
1. **Workload generation problem**
2. Generating random objects
3. Generation of complex objects
4. Quantity generation
5. Synthesis

Workload generation problem



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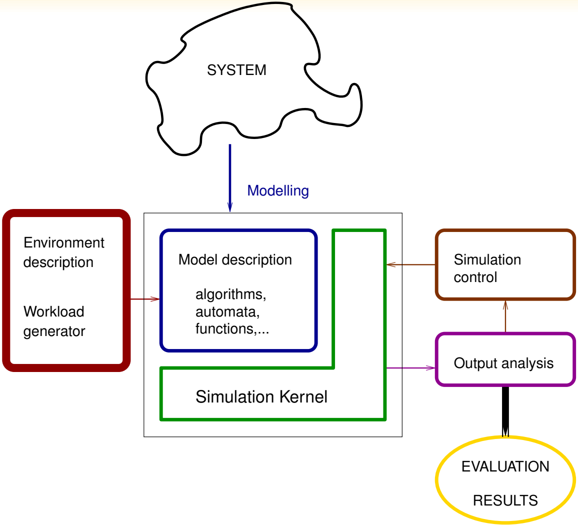
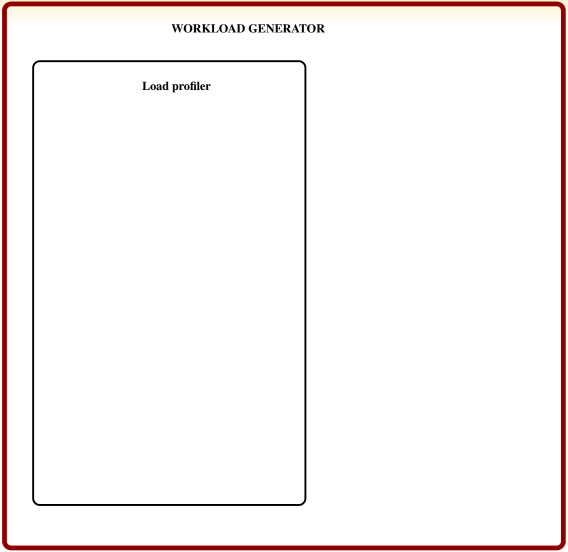


Workload generation problem (2)

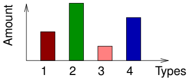


Workload generation problem (2)

WORKLOAD GENERATOR

Load profiler

Structure profile



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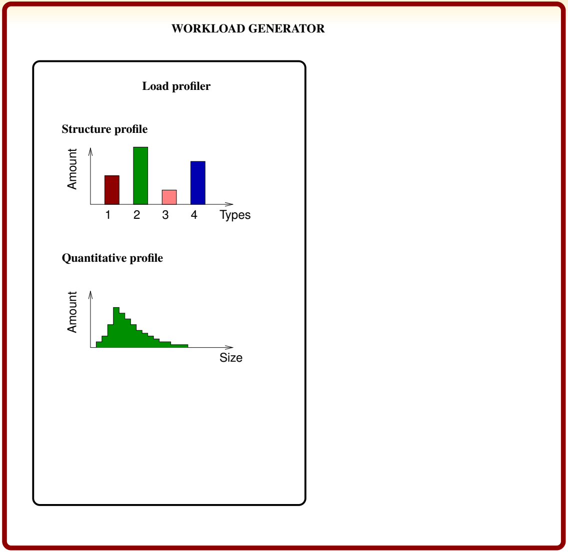

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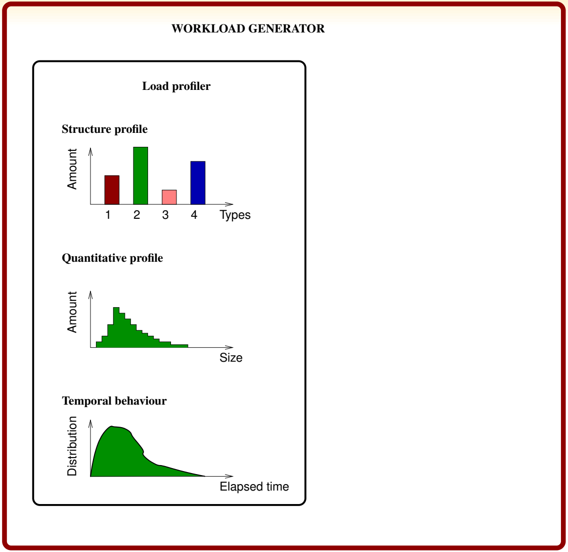

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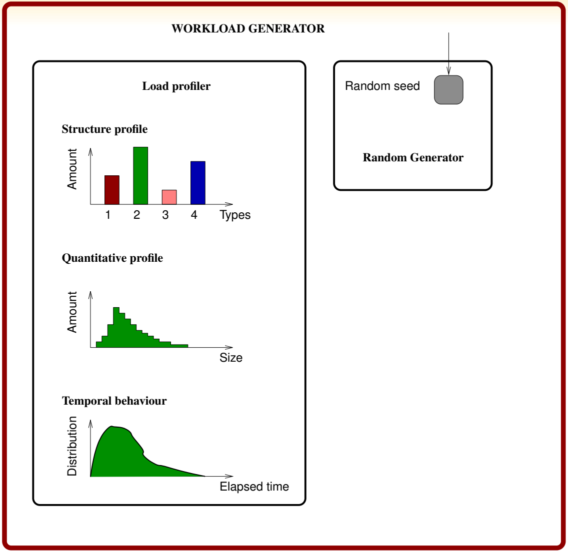

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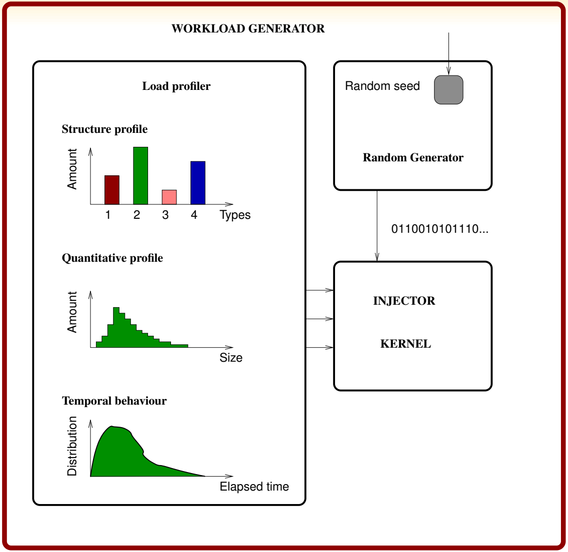

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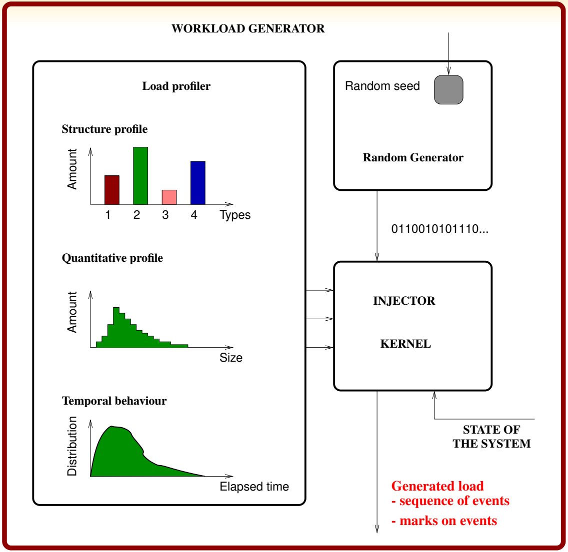

Outline

Generating random objects

Denote by X the generated object (X is a random variable)
Distribution (proportion of observations, input of the load injector)

$$p_k = \mathbb{P}(X = k).$$

Remarks :

$$0 \leqslant p_i \leqslant 1; \quad \sum_k p_k = 1.$$

Expectation (average, mean)

$$\mathbb{E}X = \sum_k k.\mathbb{P}(X = k) = \sum_k k p_k.$$

Variance and standard deviation

$$\mathbb{V}arX = \sum_k (k - \mathbb{E}X)^2 \mathbb{P}(X = k) = \mathbb{E}X^2 - (\mathbb{E}X)^2.$$

$$\sigma(X) = \sqrt{\mathbb{V}arX}.$$

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The random function

Random bit generator (see previous lecture)

drand48 manpage

double drand48(void) (48 bits encoded in 8 bytes)
The rand48() family of functions generates pseudo-random numbers using a linear congruential algorithm working on integers 48 bits in size. The particular formula employed is r(n+1) = (a * r(n) + c) mod m where the default values are for the multiplicand a = 0xfdeece66d = 25214903917 and the addend c = 0xb = 11. The modulo is always fixed at m = 2 ** 48. r(0) is called the seed of the random number generator.

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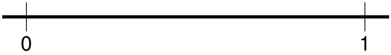

Problem

All the difficulty is to find a function (an algorithm) that transforms the $[0, 1[$ in a set with a good probability conserving.

Example : flip a coin

u= r andom()
if u $\leqslant \frac{1}{2}$ **then**
    return Head
else
    return Tail
end if

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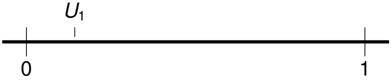

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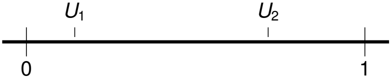

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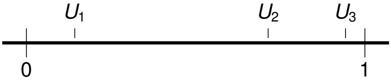

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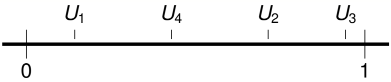

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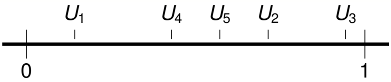

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Practical example : Web server

Types of request

1. Professional customer, consult
2. Professional customer, purchase
3. Non professional customer, consult
4. Non professional customer, purchase
5. Adminstration

Build an algorithm that provides a set of requests according the observed distribution.

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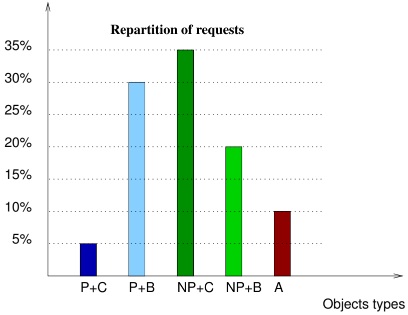

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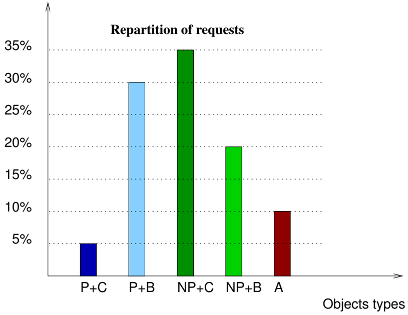

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Tabulation method

Pre-computation

$$p_k = \frac{m_k}{m} \text{ where } \sum_k m_k = m.$$

Create a table T with size m.
Fill T such that m_k cells contains k.
Computation cost : m steps
Memory cost : m

Table construction

```
i=0
for k=1, k ⩽ K, k++ do
    for j=1, j ⩽ m_k, j++ do
        T[i]= k ; i=i+1 ;
    end for
end for
```

Generation

Generate uniformly on the set $\{0, 1, \cdots, m-1\}$
Returns the value in the table
Computation cost : $\mathcal{O}(1)$ step
Memory cost : $\mathcal{O}(m)$

Generation algorithm

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u= r andom() ;
i= (int) floor(u*m)
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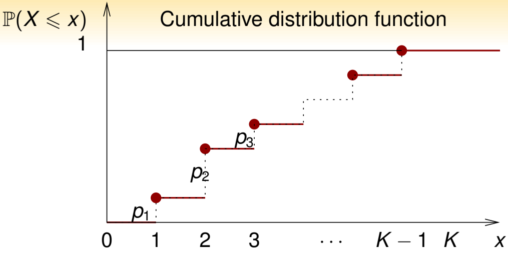

Generation

Divide $[0,1[$ in intervals with length p_k
Find the interval in which *Random* falls
Returns the index of the interval
Computation cost : $\mathcal{O}(\mathbb{E}X)$ steps
Memory cost : $\mathcal{O}(1)$

Inverse function algorithm

s=0 ; k=0 ;
u=random()
while u $>$s **do**
    k=k+1
    s=s+p_k
end while
return k

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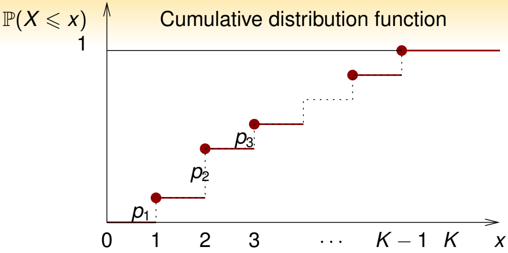

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Searching optimization

Optimization methods

- pre-compute the pdf in a table
- rank objects by decreasing probability
- use a dichotomy algorithm
- use a tree searching algorithm (optimality = Huffmann coding tree)

Comments

- Depends on the usage of the injector (repeated use or not)
- pre-computation usually $\mathcal{O}(K)$ could be huge
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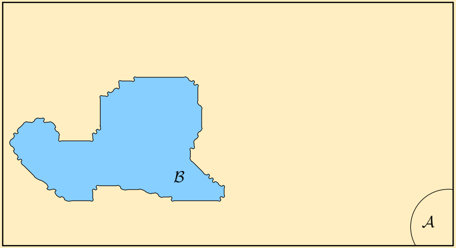
Rejection technique

Base of the method

Generate uniformly on $\mathcal{A}$ accept when point is in $\mathcal{B}$.



Rejection algorithm

repeat
  x = uniform-generate($\mathcal{A}$)
until x$\in \mathcal{B}$
return x

Complexity

Acceptance probability

$$p_a = \frac{Size(\mathcal{B})}{Size(\mathcal{A})}$$

N number of iterations

$$\mathbb{E}N = \frac{1}{p_a}.$$

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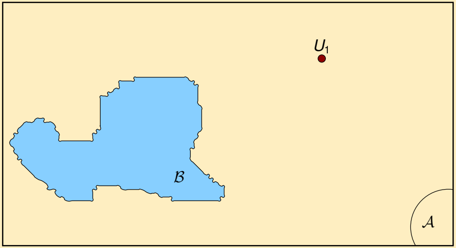

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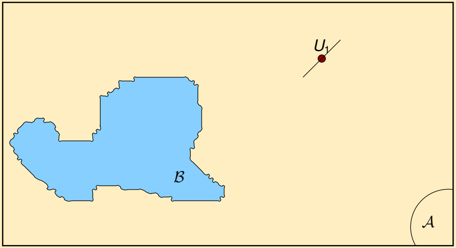

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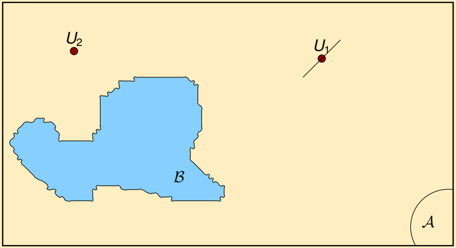

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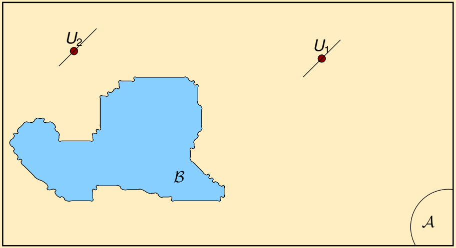

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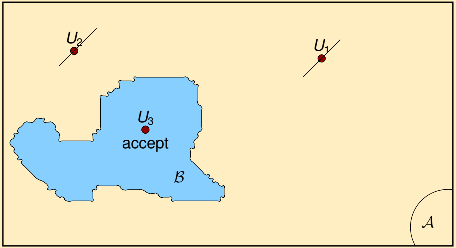

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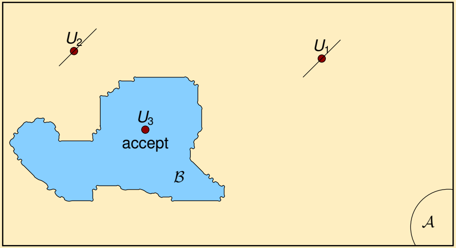

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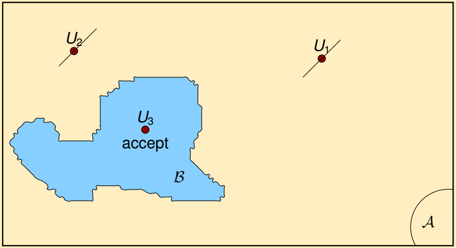

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Rejection technique

Rejection adaptation

K objects

$$h \geqslant \max_k p_k$$

Generate uniformly on the surface $K \times h$
Accept if the point is under the distribution

Rejection algorithm

repeat
  k= alea(K)
until Random . h $\leqslant p_k$
return k

alea(K) generate uniformly a number in $\{1, \cdots, K\}$

Complexity

Acceptance probability $p_a = \frac{1}{hK}$
N number of iterations $\mathbb{E}N = \frac{1}{p_a} = hK$.
Minimal complexity for $h^* = \max_k p_k$.
Uniform distribution $\Rightarrow$ no rejection
Interest : distribution near the uniform distribution

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Accept if the point is under the distribution

Rejection algorithm

repeat
  k= alea(K)
until Random . h $\leqslant p_k$
return k

alea(K) generate uniformly a number in $\{1, \cdots, K\}$

Complexity

Acceptance probability $p_a = \frac{1}{hK}$
N number of iterations $\mathbb{E}N = \frac{1}{p_a} = hK$.
Minimal complexity for $h^* = \max_k p_k$.
Uniform distribution $\Rightarrow$ no rejection
Interest : distribution near the uniform distribution

Aliasing technique

Combine uniform and alias value when rejection

Initialization

K objects

- list L=$\emptyset$,U=$\emptyset$;
- **for** k=1 ; k$\leqslant$ K ; k++ **do**
  - P[k]=p_k
  - **if** P[k] $\geqslant \frac{1}{K}$ **then**
    - U=U+$\{$k$\}$;
  - **else**
    - L=L+$\{$k$\}$;
  - **end if**
- **end for**

Alias and threshold tables

- **while** L $\neq \emptyset$ **do**
  - Extract $k \in L$
  - Extract $i \in U$
  - S[k]=P[k]
  - A[k]=i
  - P[i] = P[i] - ($\frac{1}{K}$-P[k])
  - **if** P[i] $\geqslant \frac{1}{K}$ **then**
    - U=U+$\{$i$\}$;
  - **else**
    - L=L+$\{$i$\}$;
  - **end if**
- **end while**

Aliasing technique

Combine uniform and alias value when rejection

Initialization

K objects

```
list L=∅,U=∅ ;
for k=1 ; k⩽ K ; k++ do
    P[k]=p_k
    if P[k] ⩾ 1/K then
        U=U+{k} ;
    else
        L=L+{k} ;
    end if
end for
```

Alias and threshold tables

```
while L ≠ ∅ do
    Extract k ∈ L
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    S[k]=P[k]
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```

Aliasing technique : generation

Generation

k=alea(K)
if Random . $\frac{1}{K} \leqslant$ S[k] **then**
  return k
else
  return A[k]
end if

Complexity

Computation time :
- $\mathcal{O}(K)$ for pre-computation
- $\mathcal{O}(1)$ for generation

Memory :
- threshold $\mathcal{O}(K)$ (real numbers as probability)
- alias $\mathcal{O}(K)$ (integers indexes in a tables)

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Outline

Generation of complex objects

Structured workload

- task graph
- sequence of pages
- route to destination
- ...

Structured environment

- Interconnection graph
- memory configuration
- repartition of sites on an area...
- ...

Generate uniformly a set of k positions among n possibilities

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Route generation

Given a feed-forward communication network, generate uniformly a route between two nodes

Manhattan topology

General topology

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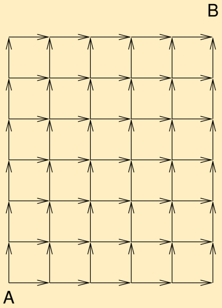

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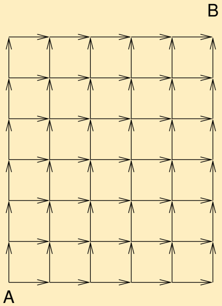

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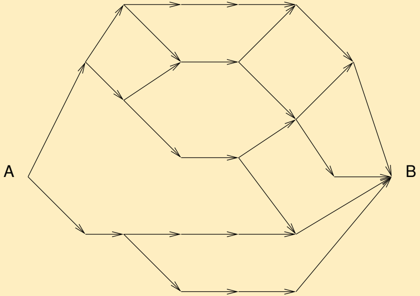

Permutation generation

Given a size N of an array generate a uniform permutation of its elements.

Based on position

for i=1 ; i⩽ N-1 ; i++ **do**
  j=alea(N-i)
  {Generate uniformly on $\{0, 1, \cdots, N-i\}$ }
  Exchange(i,i+j)
end for

Based on value

Generate_Permutation(N-1)
j=alea(N)
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for i=N ; i>j ; j- - **do**
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T[j]=N

Binary tree generation

Given a size N generate a binary tree uniformly on the set of trees with N nodes

Uniform node decomposition

Recursive algorithm

```
tree Generate_tree(integer N)
if N=0 then
   return empty_tree
else
   q=alea(0,N-1)
   TL=Generate_tree(q)
   TR=Generate_tree(N-1-q)
   T=Join(TL,TR)
   return T
end if
```

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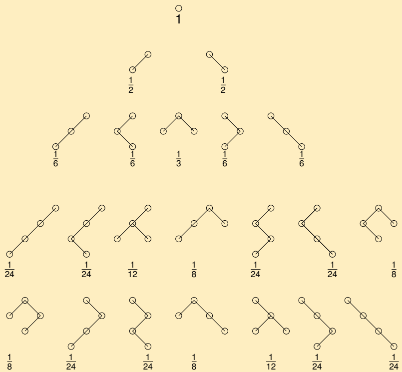

Uniform binary tree generation

Catalan's numbers

Recursion equation

$$C_0 = C_1 = 1;$$

$$C_N = \sum_{q=0}^{N-1} C_q C_{N-1-q}.$$

Then

$$1 = \sum_{q=0}^{N-1} \frac{C_q C_{N-1-q}}{C_N} = \sum_{q=0}^{N-1} p_{N,q}.$$

$$C_N = \frac{1}{N+1} \binom{2N}{N}$$

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Pre-computation of the $p_{N,q}$

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Pre-computation of the $p_{N,q}$

Outline

Generation of length, duration,...

The workload is defined by :
- type
- structure
- amount of work
- time distribution

- service duration
- communication time
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Generation of continuous variates

From a probability density, generate samples of variates in a continuous state space.

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Generating random quantities

Denote by X the object size (X is a real valued random variable)
Distribution density

$$f(x)dx = \mathbb{P}(X \in [x, x + dx[).$$

Remarks :

$$0 \leqslant f(x); \quad \int f(x)dx = 1.$$

Expectation (average, mean)

$$\mathbb{E}X = \int xf(x)dx.$$

Variance and standard deviation

$$\mathbb{V}arX = \int (x - \mathbb{E}X)^2 f(x)dx = \mathbb{E}X^2 - (\mathbb{E}X)^2.$$

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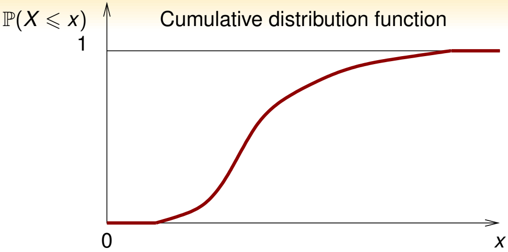
Inverse of CDF



Let $X = F^{-1}(U)$

$$\mathbb{P}(X \leqslant x) = \mathbb{P}(F^{-1}(U) \leqslant x) = \mathbb{P}(U \leqslant F(x)) = F(x).$$

Classic distribution

- Uniform on $[a, b]$: $F^{-1}(u) = a + (b - a).u$
- Exponential, rate λ : $F^{-1}(u) = \frac{1}{\lambda} \log(1 - u)$
- Pareto, Weibul,...

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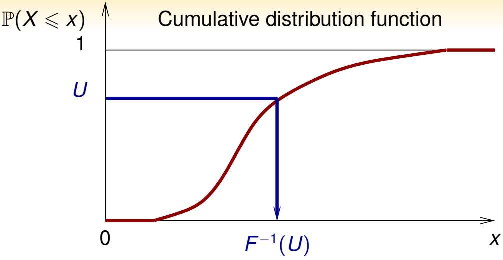

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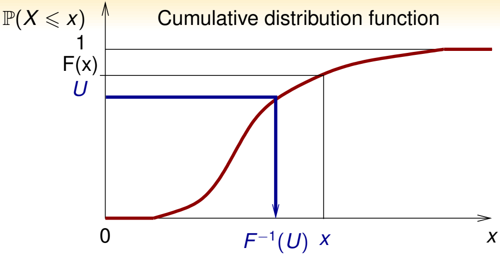

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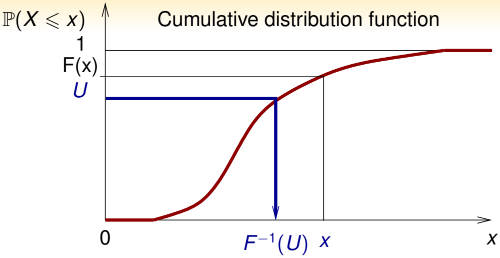

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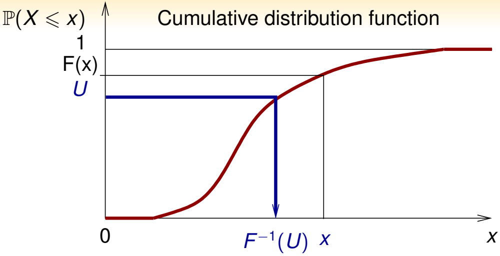

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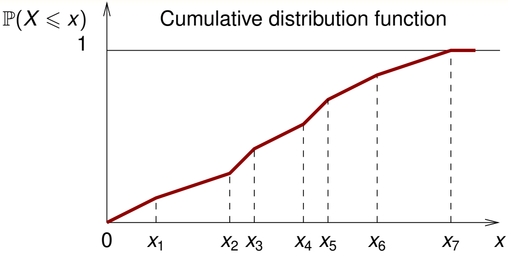
Inverse of CDF : empirical data



Set of observed values (sorted) $x_1, \cdots, x_N$, x_0 fixed by hand

```
j=alea(1,N)
x=x_{j-1} + (x_j - x_{j-1}).random
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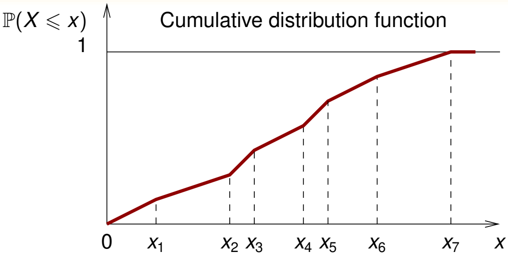
Linear interpolation Extensions : fit with middle of intervals, polynomial interpolation

Inverse of CDF : empirical data



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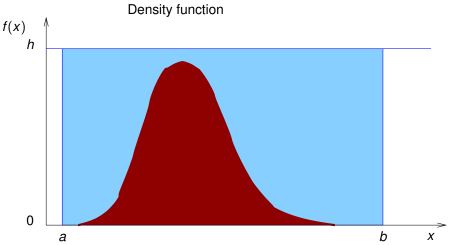
Rejection

Bounded density on a bounded interval



The rejection algorithm

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Complexity

Acceptance probability $p_a = \frac{1}{h.(b-a)}$
Mean number of iterations :
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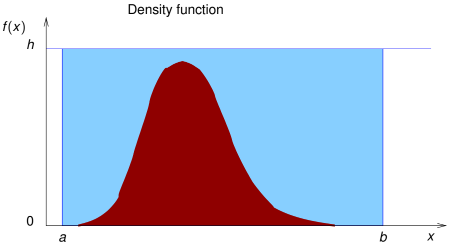

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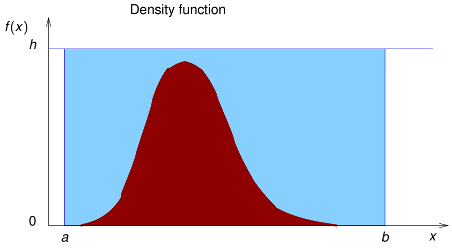

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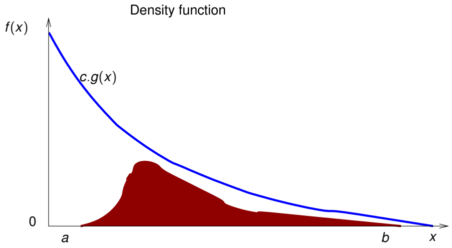
$f(x) \leqslant c.g(x)$ and there is a generator for g density



The rejection algorithm

repeat
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Complexity

Acceptance probability $p_a = \frac{1}{c}$
Mean number of iterations :
$\mathbb{E}N = c$

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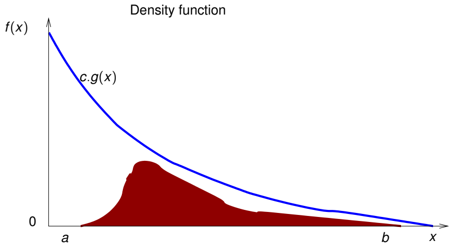

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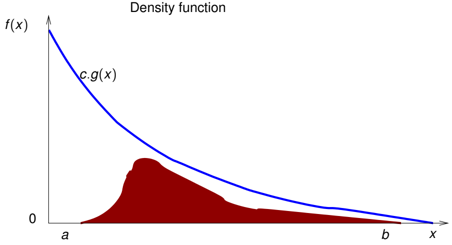

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Outline

1. Workload generation problem
2. Generating random objects
3. Generation of complex objects
4. Quantity generation
5. **Synthesis**

Synthesis

Characterization of the load :

1. Types
2. Structure
3. Quantificaton

Statistical description :

- empirical data : histograms, samples
- models (law of probability, classical distributions : uniform, exponential, erlang,...)
- bootstrapping

Validation of the workload generator : samples + statistical tests

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