Modèles pour la sécurité

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Organisation des 4 séances

1. Modèles d'attaques - Sécurité inconditionnelle, prouvée, sémantique
2. Chiffrement symétrique inconditionnellement sûrs
3. Sécurité prouvée d'un chiffrement public (RSA)
4. Générateur aléatoire cryptographiquement sûr et padding
5. Fonctions de hachage cryptographiquement sûrs
6. Protocoles à divulgation nulle de connaissance (zero-knowledge)

Ref : *Théorie des Codes : compression, cryptage, compression.* JG Dumas, JL Roch, E Tannier, S Varrette. Dunod.

Outlines

1. **Introduction : attack models and security properties**
2. Security definitions and proofs - Perfect secrecy
3. Elementary notions in probability theory
4. Shannon' theorem on perfect secrecy

Security : what cryptography should provide

CAIN

- Confidentiality
- Authentication
- Integrity
- Non-repudiation

Kerckhoffs' principle [1883]

A cryptosystem should be secure even if everything about the system, except the key, is public knowledge.

What a cryptographic protocol should provide :

- "real-time" encoding/decoding : cost$=\Theta($*message size*$)$
- "impossibility" for an attacker to decrypt an encrypted message without knowing the secret decoding function

Attack models : COA / KPA / CPA / CCA (1/2)

COA : Ciphertext-Only Attack

the attacker is assumed to have access only to a set of ciphertexts

Eg :*vulnerabilities to COA :*

- *WEP : bad design ;*
- *DES : too small key space*

KOA : Known-Plaintext Attack

the attacker has samples of both the plaintext and its encrypted version ; he uses them to get the secret key.

Eg : *vulnerabilities to KOA : encrypted ZIP archive : knowing only one unencrypted file from the archive is enough to calculate the key*

Attack models : COA / KPA / CPA / CCA (2/2)

CPA : Chosen-Plaintext Attack

the attacker chooses a plaintext and can crypt it to obtain the corresponding ciphertexts ; i.e.
he has access to an encryption machine.

Eg : *vulnerabilities to COA : dictionary attack on Unix* `passwd` *file.*
`Crack, John the Ripper, L0phtCrack, Cain&Abel, ...`

CCA : Chosen-Ciphertext Attack

the attacker chooses a ciphertext and can decrypt without knowing the key.
He has access to a decryption machine (oracle).
$\hookrightarrow$ Important for smart cards designers, since the attacker has full control on the device !

Eg : *vulnerabilities to CCA : ElGamal, early versions of RSA in SSL, ...*

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Security definitions. A cryptosystem is

Computationally secure :

if any successful attack requires at least N operations, with N large eg $10^{120} \simeq 2^{400}$.

Provable secure :

if any attack exists, a known hard problem could be efficiently solved. [Proof : reduction, complexity, P, NP]

Semantic secure – for *asymmetric cryptosystem* – :

knowing the public key and a ciphertext (COA), it must be infeasible for a computationally-bounded adversary to derive significant information about the plaintext

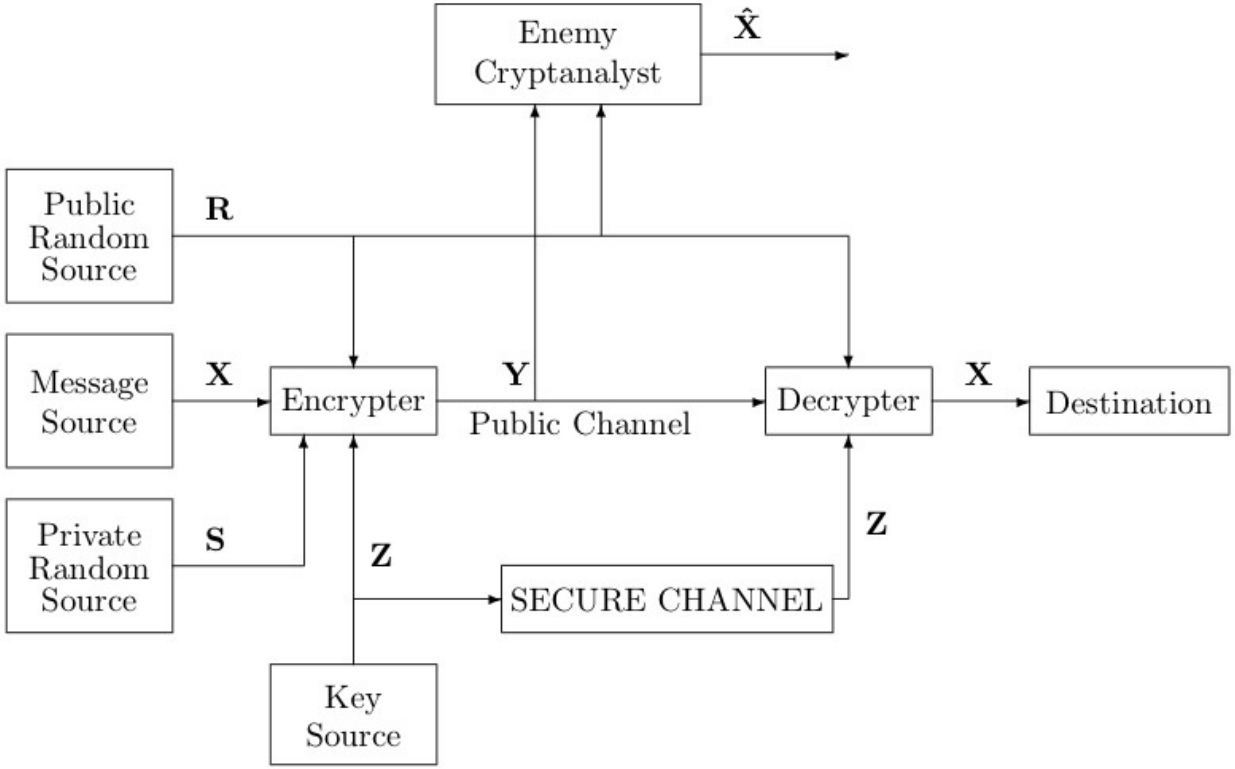
NB equivalent to the property of *ciphertext indistinguishability*[Blum,Micali]

Unconditionally secure : (or *perfect secrecy*)

cannot be broken, even by a computationally-**un**bounded attack

$\hookrightarrow$ "Information Theory" [Shannon]

Model of a symmetric cryptosystem



Shannon model

- perfect secrecy : i.e., informally,
  the knowledge of Y gives no information on X
- $\hookrightarrow$ definition : "Information Theory"

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Discrete Random Variable (1/2)

- **Sample space** S : finite set whose elements are called "elementary events"
  Eg : can be viewed as a possible outcome of an experiment
- an **event** is a subset of S.
  $\emptyset$= the *null* event S= the *certain* event
- events A and B are **mutually exclusive** iff $A \cap B = \emptyset$

- **Probability distribution**
  a function $\Pr : X \subset S \mapsto [0, 1]$ satisfying probability axioms :
  1. $\forall$ event A : $\Pr(A) \geq 0$;
  2. if A and B mutually exclusive : $\Pr(A \cup B) = \Pr(A) + \Pr(B)$
  3. $\Pr(S) = 1$

Discrete Random Variable (2/2)

Definition : Discrete Random Variable

a function X from a finite space S to the real numbers.

$\hookrightarrow$ *quantity whose values are random*

For a real number x, the event $X = x$ is $\{s \in S : X(s) = x\}$. Thus

$$\Pr(X = x) = \sum_{s \in S : X(s) = x} \Pr(s)$$

Experiment = rolling a pair of fair 6-sided dice 

- Random variable X : the maximum of the two values
- $\Pr(X = 3) = \frac{5}{36}$

Conditional probability and independence

Def : Conditional property of an event A given another event B :

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Def : Two events A and B are independent iff

$$\Pr(A \cap B) = \Pr(A).\Pr(B)$$

So, if $\Pr(B) \neq 0$, A and B independent $\iff \Pr(A|B) = \Pr(A)$

Bayes's theorem

From definition,
$\Pr(A \cap B) = \Pr(B \cap A) = \Pr(B)\Pr(A|B) = \Pr(A)\Pr(B|A)$.
This, if $\Pr(B) \neq 0$, we have :

$$\Pr(A|B) = \frac{\Pr(A)\Pr(B|A)}{\Pr(B)}$$

Birthday paradox

Birthday paradox

Let E be a set of n elements.
If $\lceil 1.18\sqrt{n} \rceil$ elements are randomly chosen in E, then the probability of a collision is larger than 50%.

Example

- There are $\simeq 365$ days in a year, so 365 birthdays possible.
- In a group of $k = 1.18\sqrt{365} \simeq 22.5$ peoples, the probaility that two persons have the same birthday is $\geq \frac{1}{2}$! !

Many applications in cryptography

block cipher (2-DES) ; collisions for hash functions, ...

Outlines

1. Introduction : attack models and security properties
2. Model of a symmetric cryptosystem
3. Elementary notions in probability theory
4. **Information theory - Shannon' theorem on perfect secrecy**

Information and entropy

Shannon's measure of information

- Hartley's measure of information : $I(X) = \log_2 \frac{1}{p_i}$ *bit* (*logon*)
- Def : entropy $H(X)$ (or uncertainty) of a disc. rand. var. X :

$$H(X) = -\sum_{x \in \mathrm{Supp}(X)} \Pr(X = x).\log_2 \Pr(X = x)$$

i.e. the "average Hartley information".

Basic properties of entropy

- Let $n =$Card(Sample space) ; then $H(X) \leq \log_2 n$
  The entropy is maximum for the uniform probability distribution Gibbs'lemma
- $H(X|Y) \leq H(X) + H(Y)$
- $H(XY|Z) = H(Y|Z) + H(X|YZ)$
- $H(X|Y) \leq H(XZ|Y)$

Unconditional security / Perfect secrecy

Characterization of perfect secrecy

The knowledge of the ciphertext Y brings no additional information on the plaintext X, i.e.

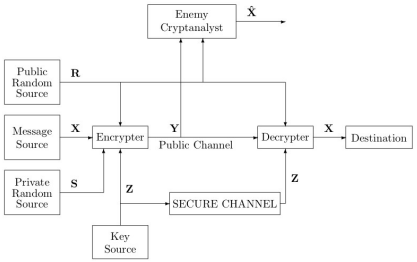
$$H(X|Y) = H(X)$$

Theorem (Shannon's theorem on perfect secrecy)

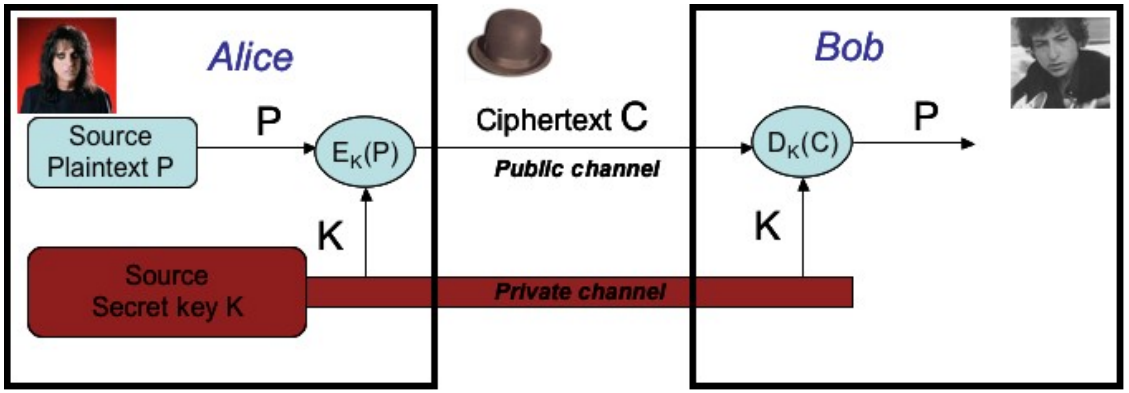
If a symmetric cryptosystem ensures perfect secrecy, then the entropy of the secret shared key Z is larger than the one of the plaintext X.

$$\textit{If } H(X|Y) = H(X) \textit{ then we have : } H(Z) \geq H(X)$$

Unconditional security of a symmetric cryptosystem



General model



Simplified model

Definition : *Unconditional security* or *Perfect secrecy*

The symmetric cipher is unconditionally secure iff $H(P|C) = H(P)$

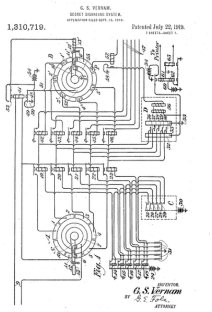
i.e. the cryptanalyst's a-posteriori probability distribution of the plaintext, after having seen the ciphertext, is identical to its a-priori distribution.

Shannon's theorem : necessary condition, lower bound on K

In any unconditionally secure cryptosystem : $H(K) \geq H(P)$.

Proof : $H(P) = H(P|C) \leq H((P,K)|C) = H(K|C) + H(P|(K,C)) = H(K|C) \leq H(K)$

Vernam's cipher : unconditionally secure cryptosystem



Vernam's patent, 1917

Symmetric cipher of a bit stream :

- let $\oplus$ = boolean *xor*; let $n = |P|$.
- for $i = 1, \ldots, n$: $\quad C_i = P_i \oplus K_i$

OTP : One-Time Pad [AT&T Bell labs]

NB : size of the (boolean) key K = size of the (boolean) plaintext P.

Theorem : Vernam's cipher is unconditionally secure

Proof :

Application One-time pad

- Unbreakable if used properly. A one-time pad must be **truly random data** and must be **kept secure** in order to be unbreakable.
- intensively used for diplomatic communications security in the 20th century. E.g. telex line Moscow–Washington : keys were generated by hardware random bit stream generators and distributed via trusted couriers.
- In the 1940s, the (Soviet Union) KGB used recycled one-time pads, leading to the success of the NSA code-breakers of the project VENONA [http ://www.nsa.gov/venona/]

Symmetric cryptography - Block ciphers

- E_K, D_K one-way mapping (bijective) $\{0,1\}^n \rightarrow \{0,1\}^n$
- block encryption : S-box + rounds + key expansion
  each round= composition of permutation and substitution
  - DES : block size=64 bits ; key size = 56 bits
  - AES : block size=128 ; key size= 128, 192 or 256
- chaining mode

Conclusion

- It is possible to provide unconditional security, but keys are too long for a practical use
- tradeoff : block cipher
- Next lecture : asymetric cryptography and provable security