Stochastic Automata Networks
&
Phase Type Distributions

I. Sbeity, L. Brenner, B. Plateau and W.J. Stewart

http://www-id.imag.fr

Motivations

- Stochastic Automata Networks (SAN): modelling very large and complex Markov chains in a campact and structured manner : tensor algebra.
- The passage of time in the evoluation of SAN components is modeled using an exponential distributions.
- The use of general distributions remains a desirable objective.
- How Phase-Type distribution can be used in SAN? the complexity of this step?..

Related works

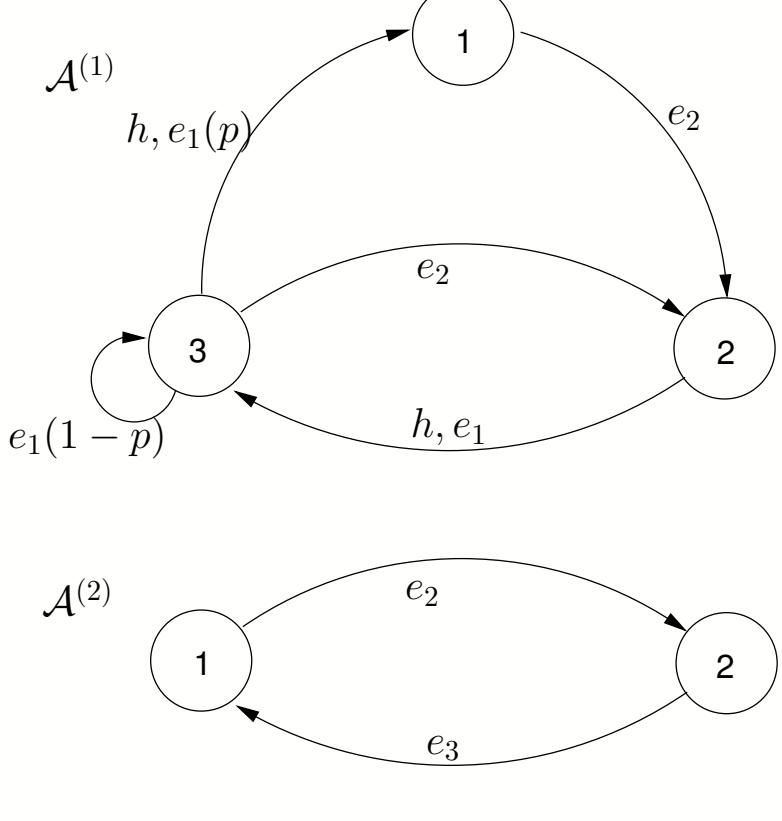
- Queueing Networks: [M.Neuts, P. Buchholz and others].
- Petri Nets:
  - The regeneration points [Ajmone Marsan and others]
    Conditions on the model
  - PH transitions are replaced by a sub-net [Molloy and others].
  - PH distributions is taken into account during the generation of the RG [Cumani].

Related works

- Petri Nets:
  - The previous methods are ineffective when the state space is large.
  - [Haddad-Moreaux-others]: A tensor formula to present the infinitesimal generator of a PH-GSPN:
    - Structural decomposition basing on the *Symmetric Structural Conflict* among transitions.
    - *Drawback*: The size of elementary matrices depends on the partition! not compact in some cases.

SAN: an informal description

- Sample example: Here, the distribution of time before the occurency of events is exponential



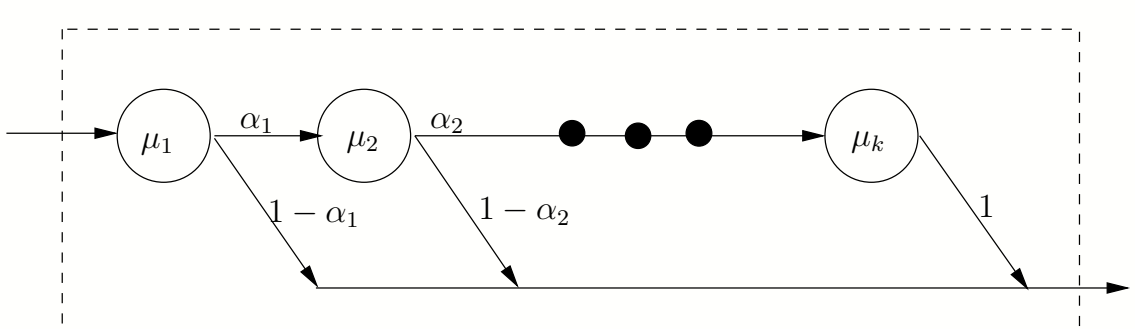
SAN: a formal description

- A SAN model is defined by $\mathcal{M} = (N, \mathcal{A}, \mathcal{E}, \hat{f})$:
  - N is the number of automata.
  - $\mathcal{A} = \{\mathcal{A}^{(1)}, \mathcal{A}^{(2)}, \ldots, \mathcal{A}^{(N)}\}$ is the set of automata.
    $\forall\, i, \mathcal{A}^{(i)} = (S^{(i)}, Edge^{(i)}, Label^{(i)})$.
    $S = S^{(1)} \times S^{(2)} \times \cdots \times S^{(N)}$ is the product state space,
    and $x = (x^{(1)}, x^{(2)}, \ldots, x^{(N)}) \in S$ is a global satate.
  - $\mathcal{E}$ is the set of events.
    $e = (loc|syn,\ master,\ \tau_e) \in \mathcal{E}$
  - $\hat{f}$ is the reachability function of the SAN .

    $\hat{S} = \{x \in S \mid \hat{f}(x) = 1\}$ is the reachable state space.

Phase Type (PH) Distribution

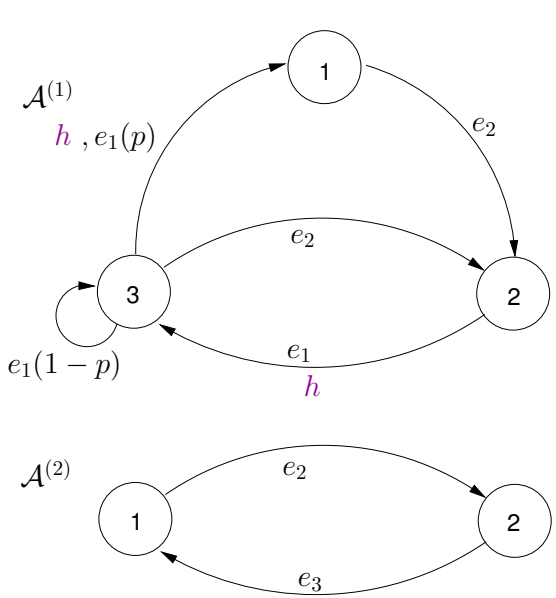
- Advantage: it permits to model more general distributions than the exponential.
- Definition: A passage through a succession of exponential phases.
- Memoryless property $\rightarrow$ 2 Preemption Policies (PP): Preempt-Resume (C) and preempt-Restart (B)
- Example: Cox distribution:

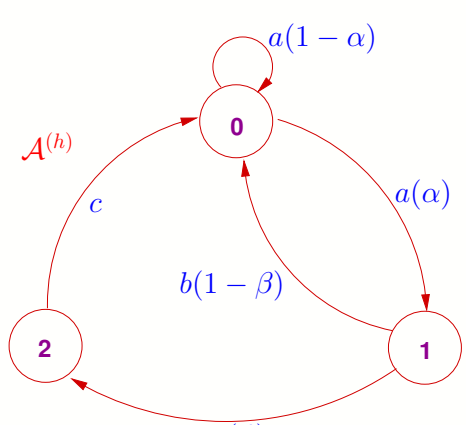


SAN with PH events

- Our Approch
  - Formal definition of a SAN model with PH distributions (PH-SAN model).
    - Each PH distribution is described by an automaton.
  - Transform a PH-SAN model to a regular SAN model by eliminating of the PH events .
    - A compact representation of the model is avoided (tensor product).
    - Matrices are small : base matrices and PH matrices.

PH-SAN model - an example





$\mathcal{E}^{(h)}$

event	rate
a	$\mu_1.f$
b	$\mu_2.f$
c	$\mu_3.f$

$f = 1,\ if\ h\ is\ fireable$

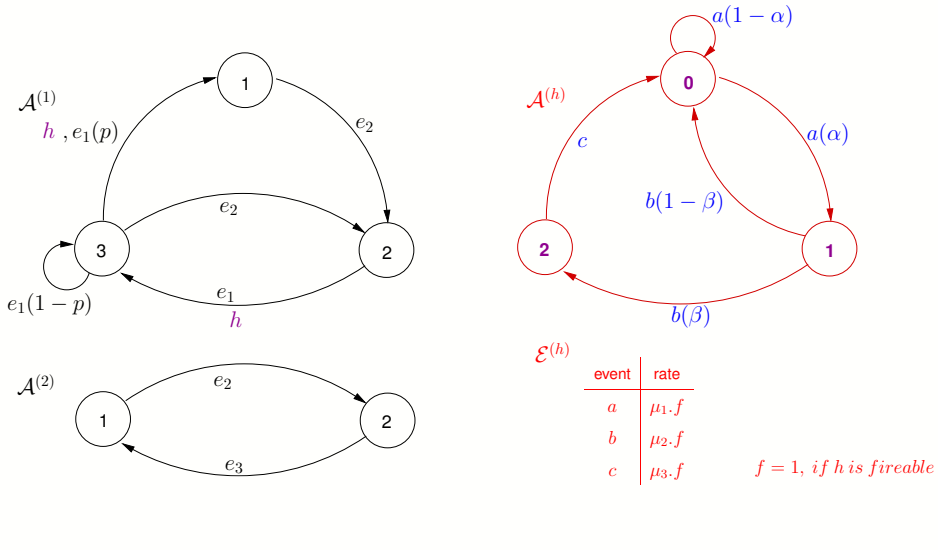
- The distribution of time before the event h fires is Cox-3.
- A description $\mathcal{D}^{(h)} = (\mathcal{A}^{(h)}, \mathcal{E}^{(h)}, PP^{(h)})$ is associated with h.

The Conflict situation

- General Definition: An enabled event e is in conflict with a enabled PH event h in a state x iff after the firing of e the PH event h is disabled (h is a censored event of e).
  - "Disabling phenomena" can be structural or functionnal.
  - "Disabling phenomena" is not symmetric.
- In the example: h is a censored event of e_1, but not for e_2.

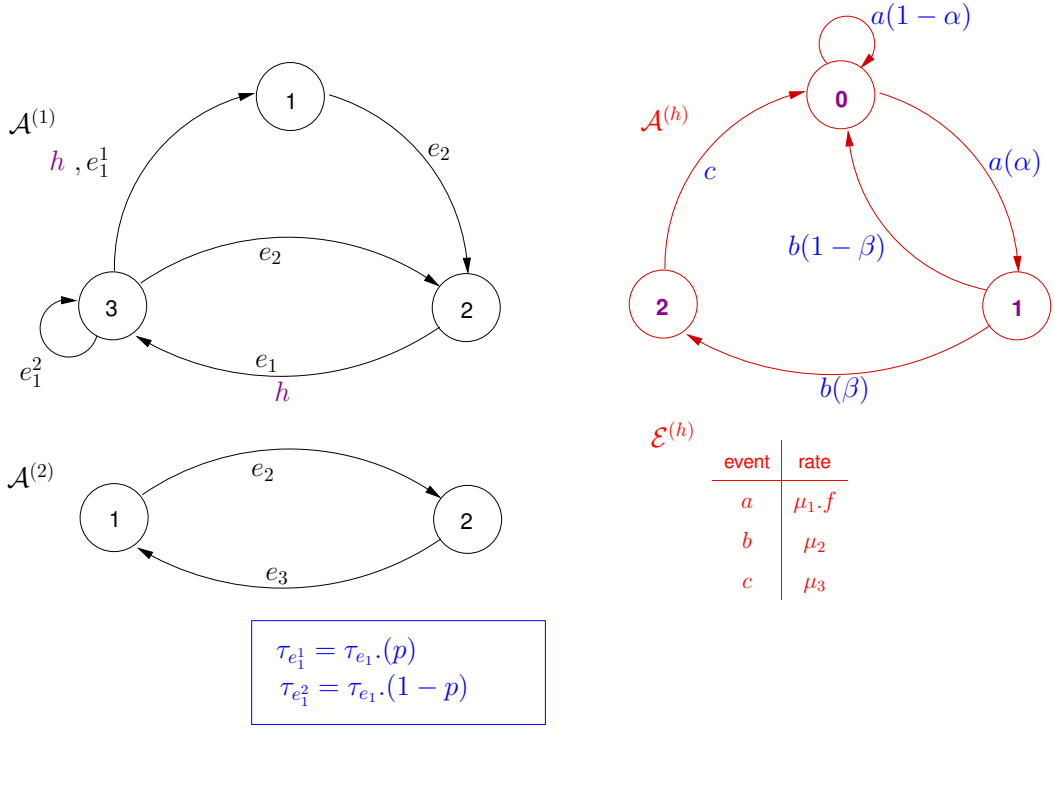
PH-SAN multiform

- An event is multiform when it has different set of censored event at the same state x (ex: event e_1).



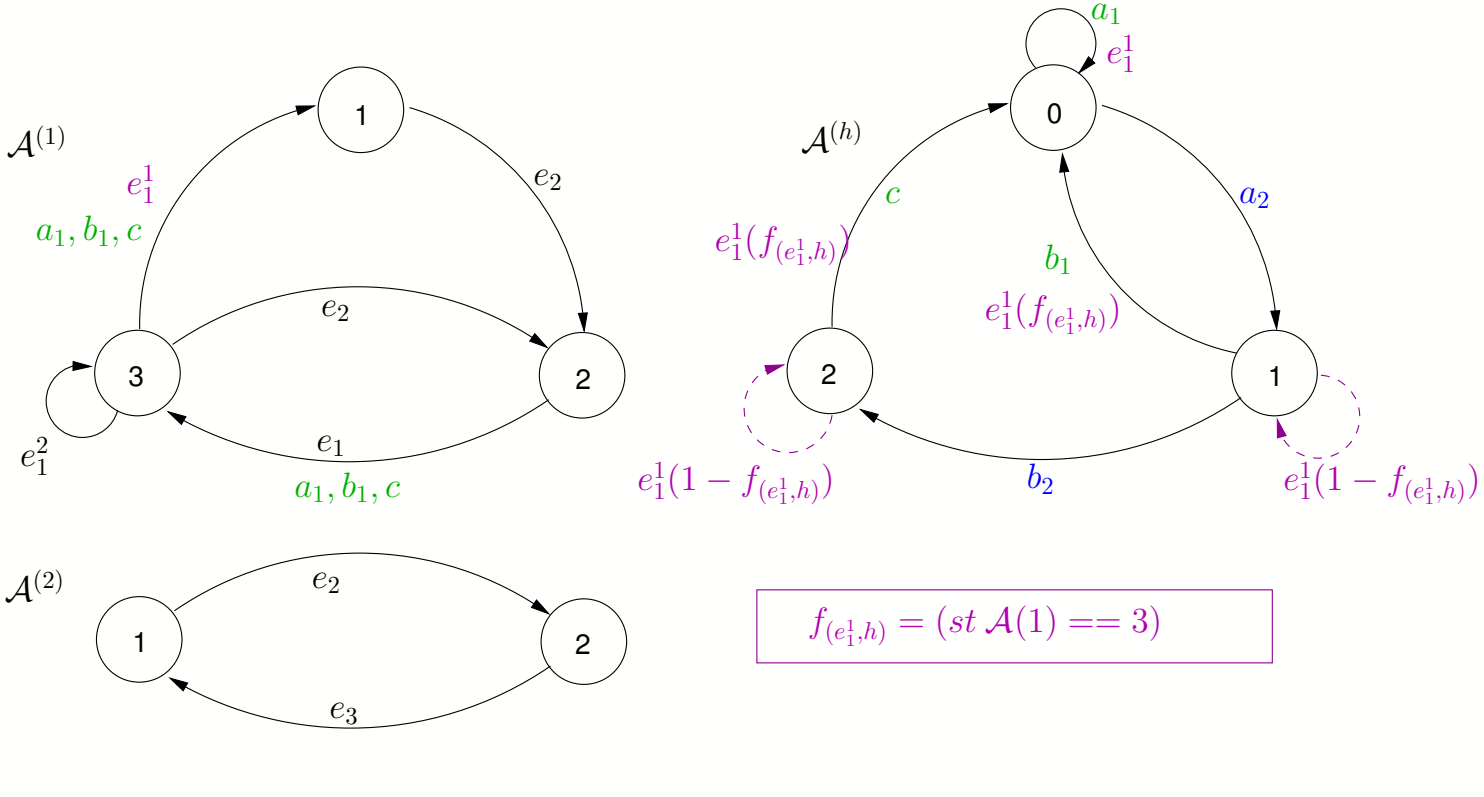
- Transforming a multiform PH-SAN to a monoform PH-SAN: decompose the multiform events.

The PH-SAN monoform



- The multiform event e_1 is replaced by two monoform events e_1^1 and e_1^2
- A PH event can be multiform !!

PH-SAN monoform to SAN



$f_{(e_1^1,h)} = (st\ \mathcal{A}(1) == 3)$

- h is replaced by the EoS events a_1, b_1 and c.
- $PP^{(h)} = B$ then e_1^1 may interrupt the work in progress of h.

Conclusions and Perspectives

- Phase-Type distributions may be incorporetad into SAN(s) formalism.
- The theorical part is done. The approach appears powerful.
- The algorithm is currently being incorporated into the PEPS software package.
- Numerical analysis.
- Development of hierarchical SAN.