Methods and algorithms for the performance analysis of large systems

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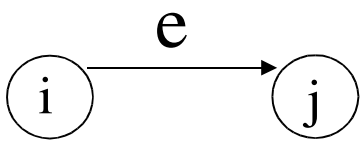
Motivations

- Continuous Time Markov Chains (CTMC): performance analysis of dynamic systems.
- Huge CTMCs: compact and structured model. State space explosion.
- Computing stationary and transient solutions: sophisticated algorithms and data structure are needed.

SAN

- Change of the global state (combination of the local states of each automata):
  - local or synchronising event, transition rate λ.
- Local event:
  - Change of the local state of only one automaton.

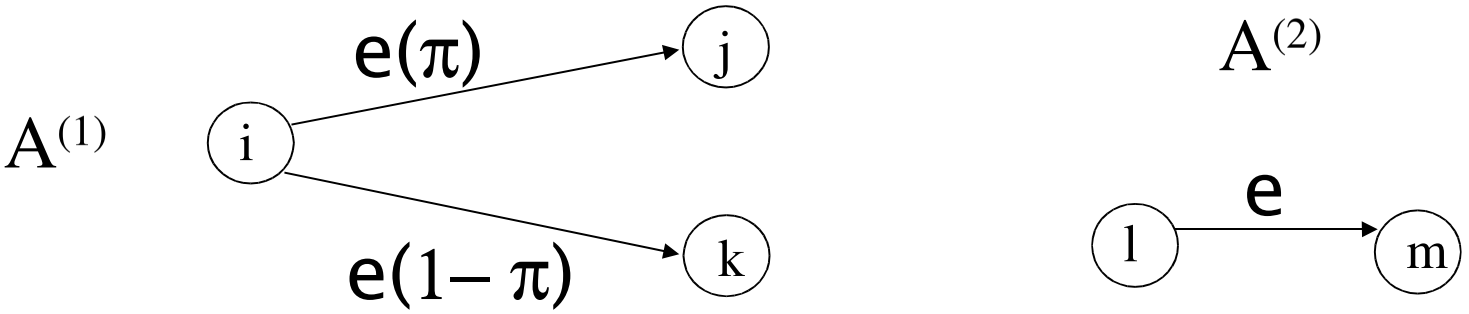


- Synchronising event:
  - Simultaneous change of the local state of more than one automaton.

SAN

- Probability of occurrence

  when the occurrence of the same event from a given departure state can lead to different arrival states.



SAN

- Other way of interacting: functional rates.
  - The transition rate of an event can be a function of the global state of the SAN.
  - Can apply to local or synchronising events.

SAN

descriptor

- SAN: modular description and manipulation. Generator of the Markov chain = descriptor Q.

$$Q = D + \sum_{i=1}^{N} {}_g Q_l^{(i)} + \sum_{e=1}^{E} \prod_{i=1}^{N} {}_g Q_e^{(i)} = \sum_{j=1}^{N+2E} \left[\prod_{i=1}^{N} {}_g Q_j^{(i)} \right]$$

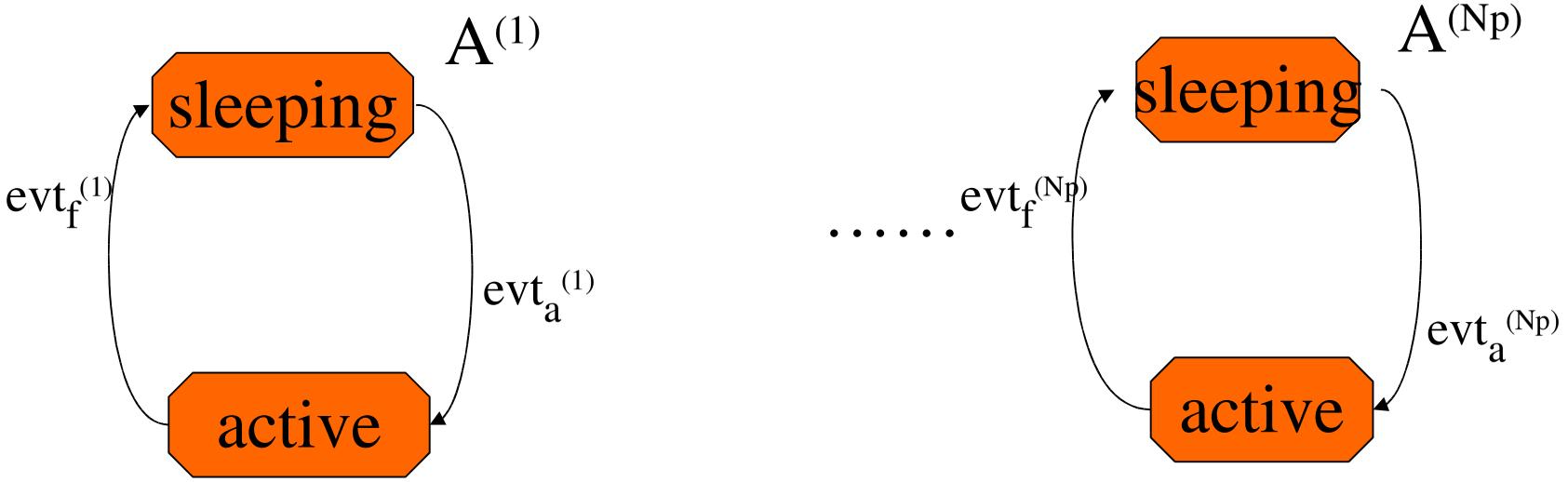
  - Tensor sum: local part.
  - Tensor product: synchronising part - E sync events.

- Use of generalised tensor algebra (functional matrices) [Plateau, Fernandes ...].

SAN

study example - mutex1

- Np processes, Nr resources - model with functions



For i=1..Np, $evt_a^{(i)}$ is a local event « acquiring a resource », rate $\lambda_i f(A^{(1)},\ldots,A^{(Np)})$.

$f(A^{(1)},\ldots,A^{(Np)})$ is a function of the number of automata in state active.

If this number is greater than or equal to Nr, f equals 0, otherwise 1.

For i=1..Np, $evt_f^{(i)}$ is a local event « freeing a resource », rate μ_i .

SAN

example - mutex1 - descriptor

- For i=1..Np, define $Q_l^{(i)} = \begin{pmatrix} -{}_i f & {}_i f \\ \mu_i & -\mu_i \end{pmatrix}$

- Then the descriptor (algebraic formulation of the transition matrix) is:

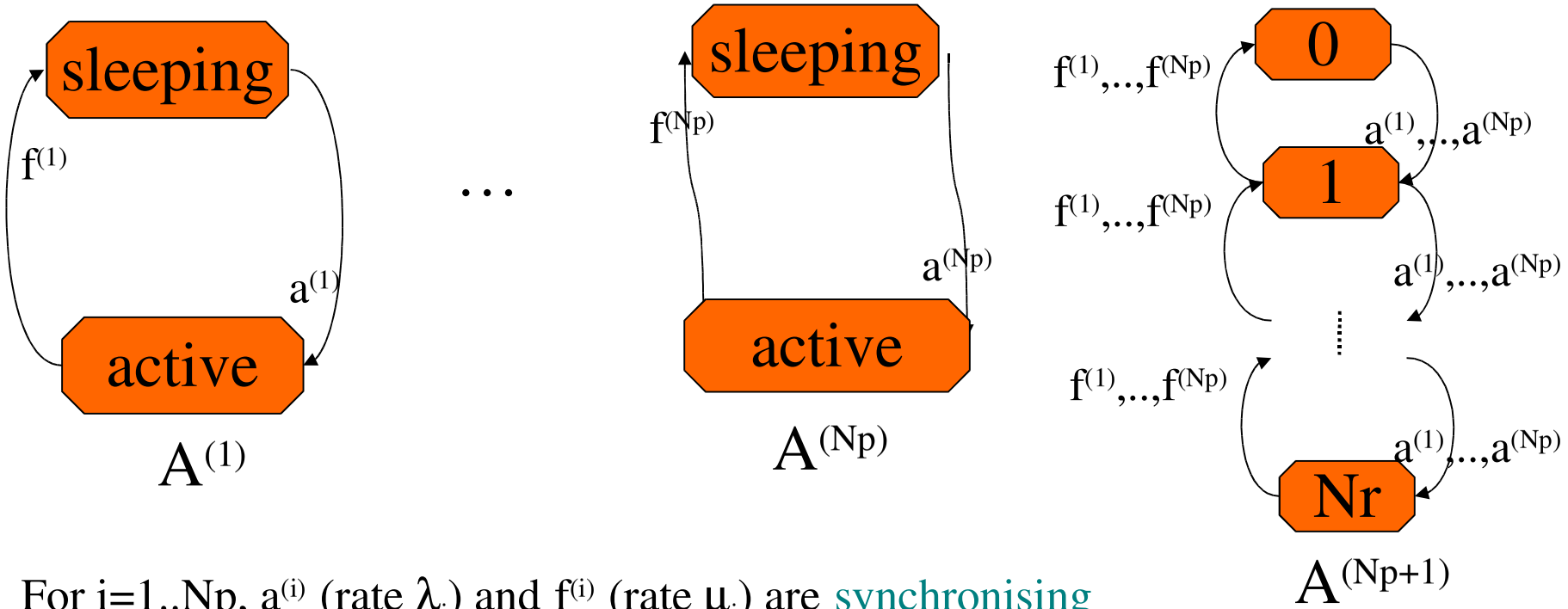
$$Q = Q_l^{(1)} \oplus_g Q_l^{(2)} \oplus_g \ldots \oplus_g Q_l^{(Np)}$$

- If the N automata were independent (i.e. the function f constant), the descriptor would be the same, with classical tensor sum.

SAN

example - mutex2

- Np processes, Nr resources - synchronising events



For i=1..Np, $a^{(i)}$ (rate λ_i) and $f^{(i)}$ (rate μ_i) are synchronising events, implying automata $A^{(i)}$ and $A^{(Np+1)}$.

SAN

comparison mutex1/ mutex2

- Product state space:

$$|\hat{S}_1| = 2^{Np} \; and \; |\hat{S}_2| = 2^{Np} * (Np + 1)$$

- Reachable state space size: $|S_1|$ and $|S_2|$

  equal to the number of solutions of

$$\sum_{i=1}^{Np} x^{(i)} \leq Nr \;, \;\; with \;\; x^{(i)} = 0 \;\; or \;\; 1$$

- Descriptor: Mutex1 - (Np-1) tensor sums.
- Mutex2 - $2(Np)^2$ tensor products of very sparse matrices.

On-going work

... and items of interest

Modelling large computer systems
formalisms

- Queuing networks
- Petri nets
- Process algebra
- Automata networks
- others:
  - UML
  - Mobius
  - combinations of PA and PN, PA and SAN?

Modelling large computer systems

storing S

- Reachability function
  - Access to the reachable states in constant time.
  - Example: PEPS.
- Multi-level approaches
  - Decision diagrams [Ciardo, Miner].
  - Efficient generation and storage of S.

Modelling large computer systems
lumping of models

reduce the complexity of the Markov chain, using replications of components and exact lumping.

- Aggregation from state space analysis
- Aggregation from the high level formalism:
  - [Siegle], [Buchholz]
  - [Benoit]: SANs with replicas and tensor structure of the aggre
  - extension and combination

Modelling large computer systems
numerical algorithms

- Context: iterative methods
- tensor: work on a product state space $\hat{S}$
- Probability vectors $\hat{}$ « extended » (the size of $\hat{S}$)
- Q: descriptor of the SAN $Q = \sum_{j=1}^{N+2E} \left[\bigotimes_{i=1}^{N} Q_j^{(i)} \right]$

- sparse implementation, in parallel?

PEPS to developp

- All these algorithms are implemented in PEPS

  (Performance Evaluation of Parallel Systems)
  - SAN editor
  - Describes easily SANs with replicas, and replicated states with identical behavior (queuing networks).
  - Computes and tests the descriptor.
  - Implements the basic multiplications, with optimizations for the function evaluation (automata reordering, ...).
  - Various iterative methods (power, Jacobi, GMRES).
  - Allows to output sparse format (HBF).
  - Compute performance indices.

Other Perspectives

- Graphical interface in Java for PEPS: more conviviality, easy to use [Mathieu Le Coz]
- Discrete-time SANs: we are working on a new formalism; develop new numerical methods.
- Compute bounds
- Simulation ?

Annexe (1)

tensor product

Product $A \otimes B =$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \otimes \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$=$

$$\left(\begin{array}{ccc|ccc} a_{11}b_{11} & a_{11}b_{12} & a_{11}b_{13} & a_{12}b_{11} & a_{12}b_{12} & a_{12}b_{13} \\ a_{11}b_{21} & a_{11}b_{22} & a_{11}b_{23} & a_{12}b_{21} & a_{12}b_{22} & a_{12}b_{23} \\ a_{11}b_{31} & a_{11}b_{32} & a_{11}b_{33} & a_{12}b_{31} & a_{12}b_{32} & a_{12}b_{33} \\ \hline a_{21}b_{11} & a_{21}b_{12} & a_{21}b_{13} & a_{22}b_{11} & a_{22}b_{12} & a_{22}b_{13} \\ a_{21}b_{21} & a_{21}b_{22} & a_{21}b_{23} & a_{22}b_{21} & a_{22}b_{22} & a_{22}b_{23} \\ a_{21}b_{31} & a_{21}b_{32} & a_{21}b_{33} & a_{22}b_{31} & a_{22}b_{32} & a_{22}b_{33} \end{array}\right)$$

Annexe (2)

tensor sum

$$\text{Sum } A \oplus B = A \otimes Idn_B + Idn_A \otimes B$$

$$= \left(\begin{array}{ccc|ccc} a_{11}+b_{11} & b_{12} & b_{13} & a_{12} & & \\ b_{21} & a_{11}+b_{22} & b_{23} & & a_{12} & \\ b_{31} & b_{32} & a_{11}+b_{33} & & & a_{12} \\ \hline a_{21} & & & a_{22}+b_{11} & b_{12} & b_{13} \\ & a_{21} & & b_{21} & a_{22}+b_{22} & b_{23} \\ & & a_{21} & b_{31} & b_{32} & a_{22}+b_{33} \end{array}\right)$$

Annexe (3)

generalised tensor product

$$\begin{pmatrix} a_{11}(k) & a_{12}(k) \\ a_{21}(k) & a_{22}(k) \end{pmatrix} \otimes_g \begin{pmatrix} b_{11}(h) & b_{12}(h) & b_{13}(h) \\ b_{21}(h) & b_{22}(h) & b_{23}(h) \\ b_{31}(h) & b_{32}(h) & b_{33}(h) \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc}
a_{11}(1)b_{11}(1) & a_{11}(1)\,b_{12}(1) & a_{11}(1)\,b_{13}(1) & a_{12}(1)b_{11}(1) & a_{12}(1)\,b_{12}(1) & a_{12}(1)\,b_{13}(1) \\
a_{11}(2)(1) & a_{11}(2)(1) & a_{11}(2)(1) & a_{12}(2)b_{21}(1) & a_{12}(2)b_{22}(1) & a_{12}(2)b_{23}(1) \\
a_{11}(3)b_{31}(1) & a_{11}(3)b_{32}(1) & a_{11}(3)b_{33}(1) & a_{12}(3)b_{31}(1) & a_{12}(3)b_{32}(1) & a_{12}(3)b_{33}(1) \\
\hline
a_{21}(1)b_{11}(2) & a_{21}(1)\,b_{12}(2) & a_{21}(1)\,b_{13}(2) & a_{22}(1)b_{11}(2) & a_{22}(1)\,b_{12}(2) & a_{22}(1)\,b_{13}(2) \\
a_{21}(2)(2) & a_{21}(2)(2) & a_{21}(2)(2) & a_{22}(2)(2) & a_{22}(2)(2) & a_{22}(2)(2) \\
a_{21}(3)b_{31}(2) & a_{21}(3)b_{32}(2) & a_{21}(3)b_{33}(2) & a_{22}(3)b_{31}(2) & a_{22}(3)b_{32}(2) & a_{22}(3)b_{33}(2)
\end{array}\right)$$