A MAGICAL SEQUENCE

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# Introduction

The objective of this work is to *study* a sequence of integers. Studying means discovering and proving some properties that highlight particular behavior of the successive terms.

## Presentation of the problem

Let call $M_n$ the n-th element of the *Magical* following bilinear sequence where the elements are computed two by two.

- $M_0 = 0$ and $M_1 = 1$
- $M_{2n} = M_n$
- $M_{2n+1} = M_{n+1} + M_n$

## Get an insight

The first elements are:

$1, 1, 2, 1, 3, 2, 3, 1, 4, 3, 5, 2, 5, 3, 4, 1, 5, 4, 7, 3, 8, 5, 7, 2, 7, 5, 8, 3, 7, 4, 5, 1, \ldots$.

This is not really a *question* but a way to enter into this problem.

Play with the first elements of the sequence and give several representations, in particular the representation where the elements are listed one after the other by rows of groups of $2^k$ on row $k$ (all rows are starting by a "1").

## Basic properties

Check and prove the following properties.

- The sum of the element of a row is a power of 3.
- The term in a given column is an arithmetic progression.
- The terms in a row are arranged in a symmetric order and more precisely, like a palindrom.
- Is there a link with the Pascal's triangle?
- Extract the maximum number on each row. What are you remarking?

It may be difficult to prove some of these properties, I am not waiting for all the answers...

## Link with rational numbers

The most remarkable property is the use of this progression to enumerate all the rational numbers[1].

Give an adequate representation and show how to get all rationals.

[1] may be with some redundoncies