

Computing a summation involving binomial coefficients

$$\text{Let } Q_n = \sum_{k \leq 2^n} (-1)^k \times \binom{2^n - k}{k}$$

Question

We are looking for an expression of Q_n

Compute $Q_{1,000,000}$

Hint

- Compute the first ranks (for $n = 1, 2, 3$ and 4).
- As binomial coefficients are involved, an idea could be to use Pascal's triangle and recurrence.
 However, $2^n - k$ is 2^k rows apart k , thus, we can not easily use the *locality character* of the triangle.
 Instead, we change by computing $R_m = \sum_{k \leq m} (-1)^k \times \binom{m-k}{k}$
 by using the relation $\binom{m-k}{k} = \binom{m-(k+1)}{k} + \binom{m-(k+1)}{k-1}$
- Compute the first ranks and show $R_m = R_{m-6}$
- Deduce the expression of $Q_{1,000,000}$