

Name:

Maths

Question N.1:

Finite summations: let n be an integer, $\sum_{k=0, n} 2^k = ?$

Question N.2:

Prove $\sum_{k=1, n} (k^2(k+1) - k(k-1)^2) = n^2(n+1)$

Question N.3:

Identities:

$$a^n - b^n = ?$$

$$(a+b)^n = ?$$

Question N.4:

What are the values of $\sum_{k>0} \frac{1}{2^k}$ and $\sum_{k>0} \frac{1}{k}$?

Question N.5:

Classify asymptotically the functions (variable n integer).

$$\log(n), 2^n, \sqrt{n}, n^n, \log(\log(n)), n^3$$

Question N.6:

Consider $T = 1 + 2 + 4 + \dots$

Compute $2T = 2 + 4 + 8 + \dots = T - 1$, thus $T = -1$.

What's wrong here?

Give an interpretation of the sum: $1-1+1-1+1 \dots$

Question N.7:

What is an irrational number?

Question N.8:

Recall the definition of a *function* $F : S \rightarrow T$.
What is an injective function?

Question N.9:

Give the definition of the derivative of the continuous function f defined on all the real points.
Describe briefly its geometric interpretation.

Question N.10:

What are the derivative of each function: $x^2 + 2x$, \sqrt{x} , $\log(x)$, $\frac{1}{x}$

Question N.11:

Recall the interpretation of the integral of a function.
Examples for $(x + 1)^2$ on $[0..1]$ and $1/x^c$ on $]0, \infty[$ for $c > 0$

Question N.12:

Consider a –continuous– function $f(x)$. Give a definition and an example for the following asymptotic notations: $O(f(x))$, $\Omega(f(x))$, $\Theta(f(x))$.

Question N.13:

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 3 & 1 \\ 0 & 5 & 2 \end{pmatrix}$$

Compute the determinant of A and compute A^2 .

Question N.14:

Write the number 2021 in base 2 (binary) and in base 16.

Question N.15:

Describe the main component of the Algebra of Propositional Logic

Question N.16:

Truth tables.
Build the table for the main operations of propositional logic.
Check the contraposition operation using truth tables.

Question N.17:

Define the notion of *equivalence relation*.

Question N.18:

Do you know the notion of *algebraic closure*?

Question N.19:

Prove that the following relation between pairs of integers (n_i, m_i) : $(n_1, m_1)\rho(n_2, m_2)$ iff $n_1 + m_2 = n_2 + m_1$ is an equivalence relation.

Give an interpretation of the equivalent class that contains $(n = 1, m = 0)$.

Question N.20:

What is an order relation?

Question N.21:

Give the formal definition of the intersection and union of two sets S and T .

Question N.22:

Give the formal definition of the set difference of S and T .

Question N.23:

Define the cross product (or cartesian product) of two sets S and T .

Question N.24:

Express $\log_a(x)$ with logarithms in base b .

Question N.25:

Give another expression for $n^{\log_a(b)}$.