The Use of Game Theory for Resource Sharing in Large Distributed Systems

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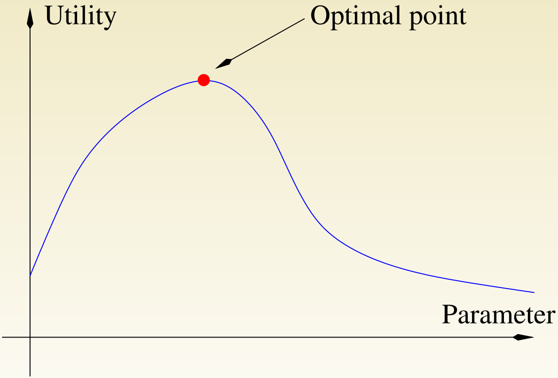
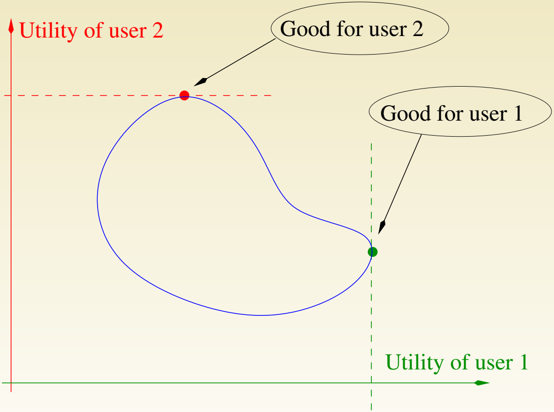
Optimality of Multi-user system

- Optimality of a single user



Optimality of Multi-user system

- Situation with multiple users



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A point is Pareto optimal if it cannot be strictly dominated

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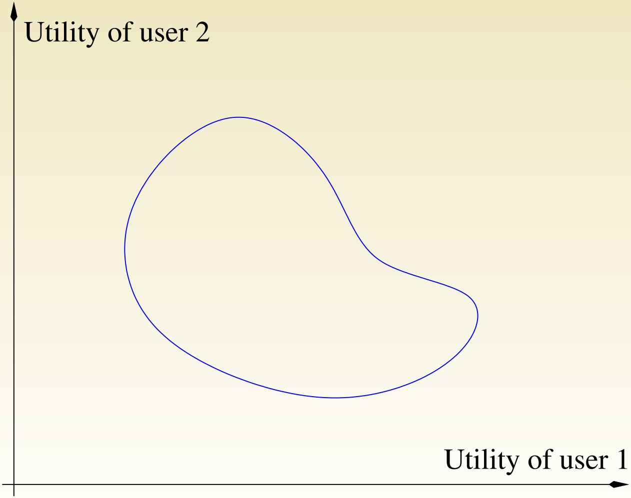

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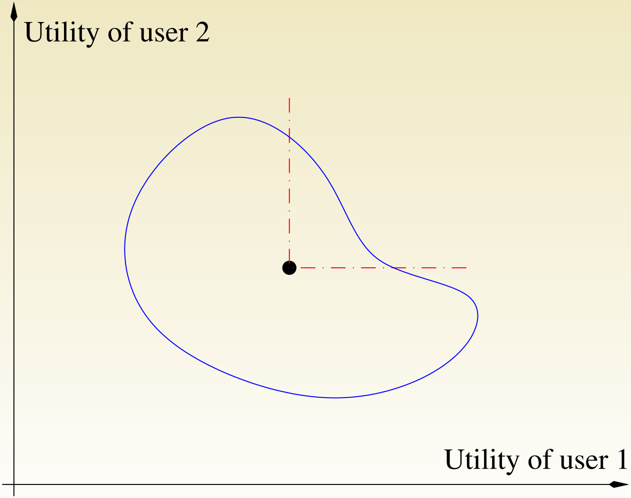

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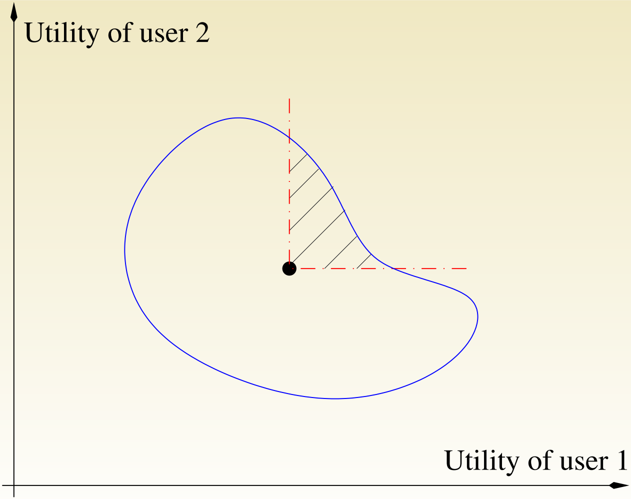

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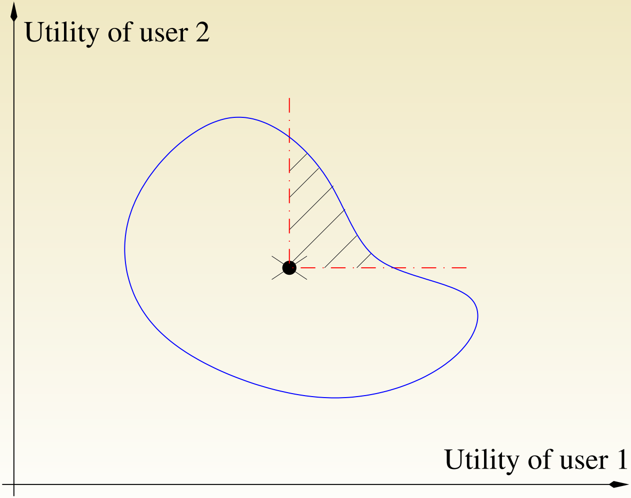

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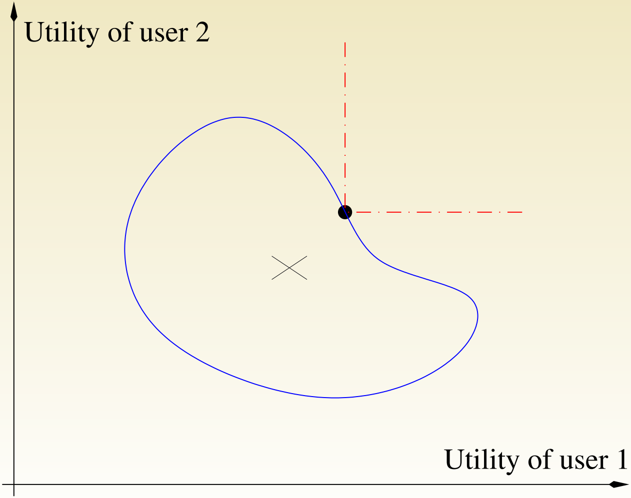

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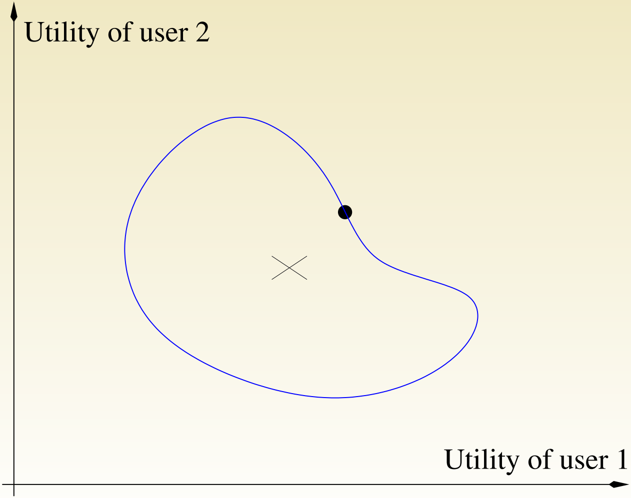

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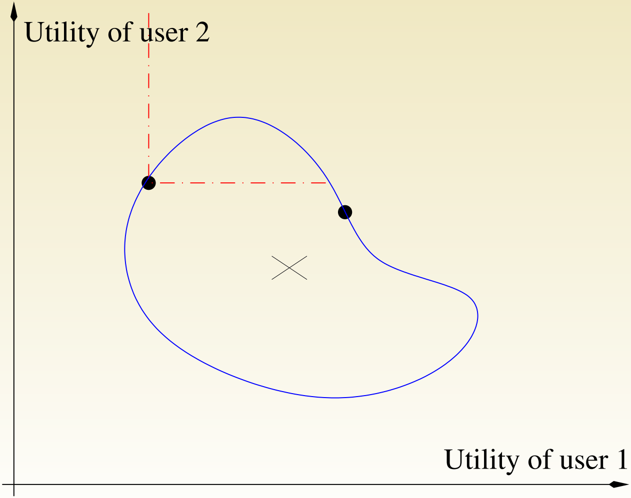

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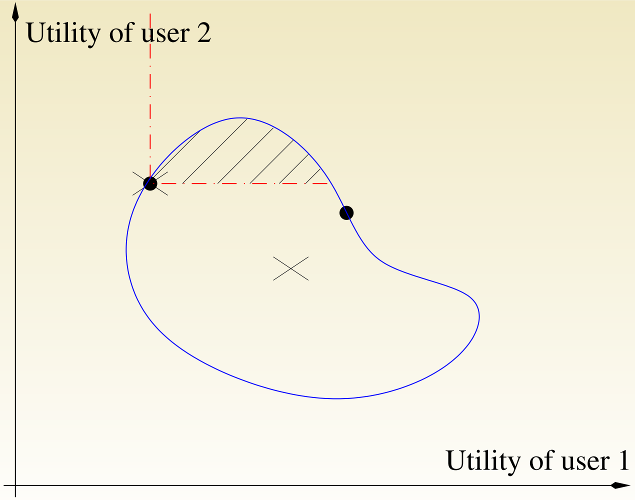

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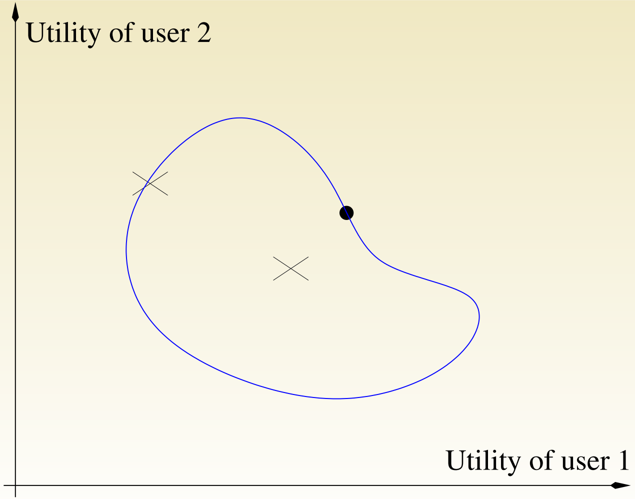

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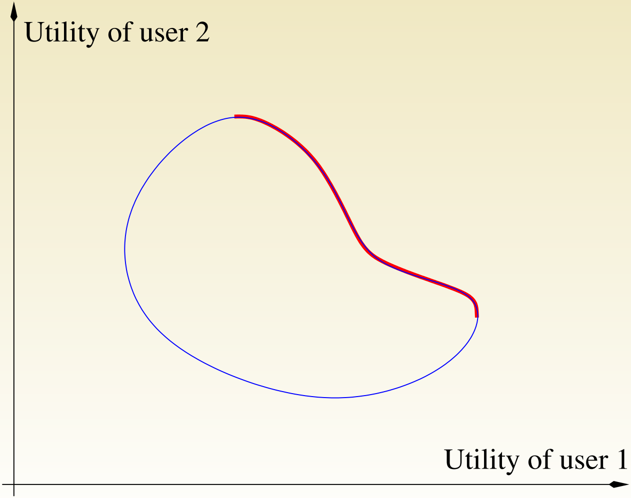

Cooperating or being selfish?

Cooperative games	Non-cooperative games
Institution setting rules and penalties to inforce them	Individual behavior converge (or not) to an equilibrium

Example: Routing intersection:

- Cooperative approach: set of roadsigns (traffic lights, "stop signs"...) inforced by the police
- Non-cooperative approach: everyone tries to cross it as quickly as possible

Outline

1. Non-cooperative optimization
   - Nash Equilibria
   - Braess Paradoxes
   - Solutions
2. Cooperative Games
   - Definitions of fairness
   - Examples
   - Non-convex systems

Outline

Definition

In a non-cooperative setting, each player makes a decision so as to maximize its own return.

Nash equilibria

In a Nash equilibrium, no player has incentive to unilaterally modify his strategy.

strategy (choice) utility

s^* is a Nash equilibrium iff:

$$\forall p, \forall s_p, u_p(s_1^*, \dots, s_p^*, \dots s_n^*) \geqslant u_p(s_1^*, \dots, s_p, \dots, s_n^*)$$

Nash Equilibria: definition (cont.)

Pros

- Intuitive

Cons

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Nash Equilibria: definition (cont.)

Pros

- Intuitive
- Easy to implement

Cons

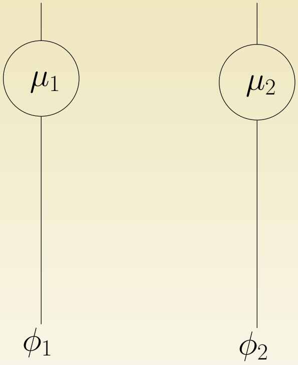
- No guaranty of existence / unicity
- difficult to compute analytically (fixed points)
- usually not Pareto optimal

Performance measure: delay

1 single server: $T = \dfrac{1}{\mu - \phi}$ $(\phi < \mu)$

Nash equilibria: Applications

1st example: A Load Balancing System [TIK04]



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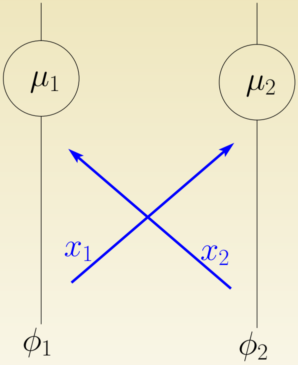

Performance measure: delay

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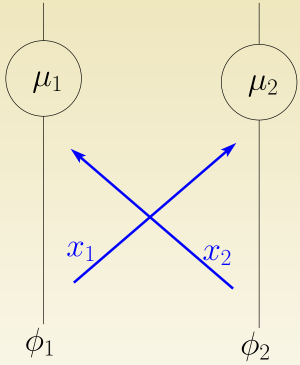
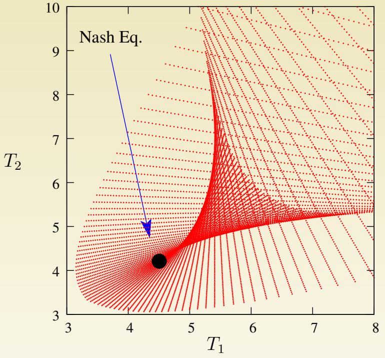
$$T_i(\boldsymbol{x}) = \frac{1}{\phi_i}\left[\frac{\phi_i - x_i}{\mu_i - \phi_i + x_i - x_j} + x_i t + \frac{x_i}{\mu_j - \phi_j + x_j - x_i}\right].$$





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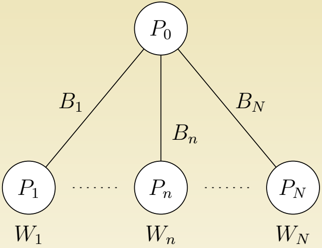
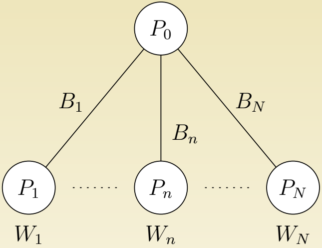


- N processors with processing capabilities W_n (in Mflop.s^{-1})
- using links with capacity B_n (in Mb.s^{-1})

Hypotheses :



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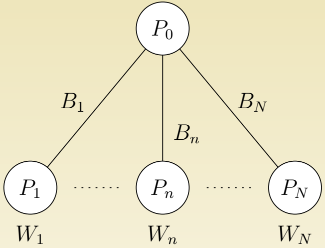
Hypotheses :

- Multi-port

Communications to P_i do not interfere with communications to P_j.

2nd example: Scheduling of bag-of-task applications [LT07]



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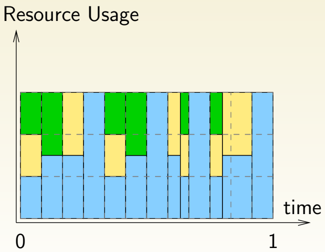
Hypotheses :

- Multi-port
- No admission policy but an ideal local fair sharing of resources among the various requests



Nash equilibria: Applications

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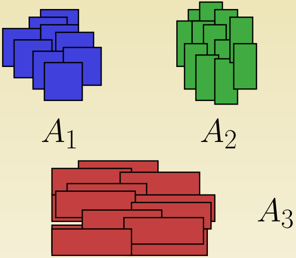

- Bag-of-tasks applications $(A_1, \dots, A_K)$
- Different needs for different applications:
  - processing cost w_k (MFlops)
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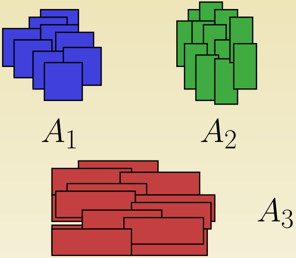

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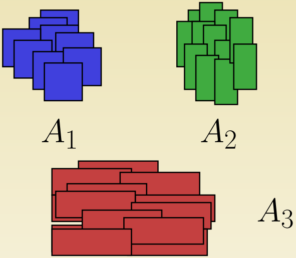

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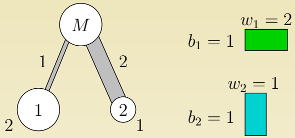
- Such applications are typical desktop grid applications (SETI@home, Einstein@Home, ...)

The K applications decide when to send data from the master to a worker and when to use a worker for computation so as to maximize their throughput (utility) α_k,

$$\alpha_k = \sum_n \alpha_{n,k}.$$

Two computers: $B_1 = 1$, $W_1 = 2$, $B_2 = 2$, $W_2 = 1$.
Two applications: $b_1 = 1$, $w_1 = 2$, $b_2 = 2$ and $w_2 = 1$.



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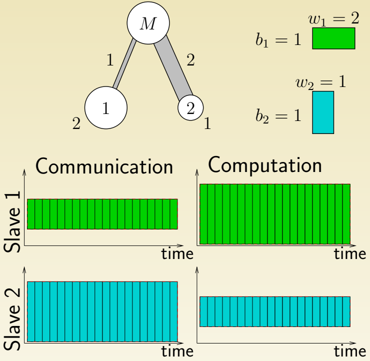

Cooperative Approach:

Application 1 is processed exclusively on computer 1 and application 2 on computer 2. Then, $\alpha_1^{(\text{coop})} = \alpha_2^{(\text{coop})} = 1$.

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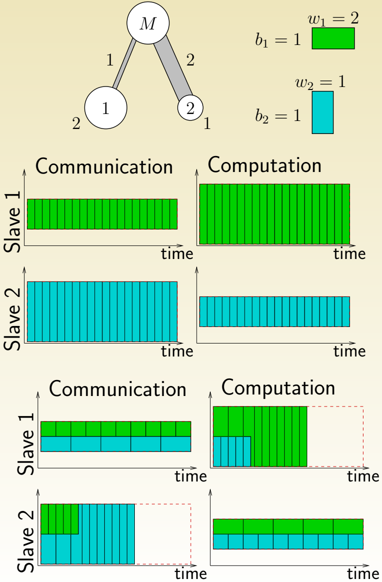

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Non-Cooperative Approach:

$$\alpha_1^{(nc)} = \alpha_2^{(nc)} = \frac{3}{4}$$

Outline

Braess Paradoxes: definition

Pareto-inefficient equilibria can exhibit unexpected behavior.

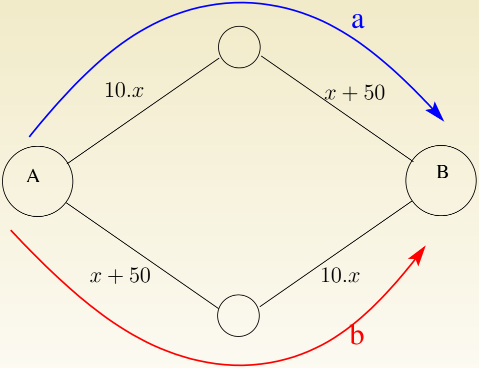
Definition: Braess Paradox.

There is a Braess Paradox if there exists two systems ini and aug such that

$$ini < aug \text{ and } \alpha^{(nc)}(ini) > \alpha^{(nc)}(aug).$$

i.e. adding resources to the system may reduce the performances of **ALL** players simulateously.

Context: urban transportation networks.
Hypothesis: travelers select their routes of travel from an origin to a destination so as to minimize their own travel cost or travel time.



Rate: 6

With 2 roads,
$Cost_a = Cost_b = 83$

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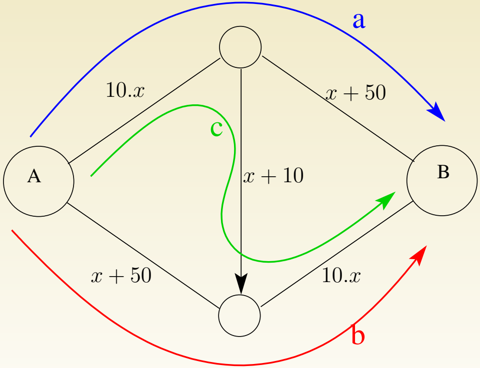


Rate: 6

With 2 roads,
$Cost_a = Cost_b = 83$

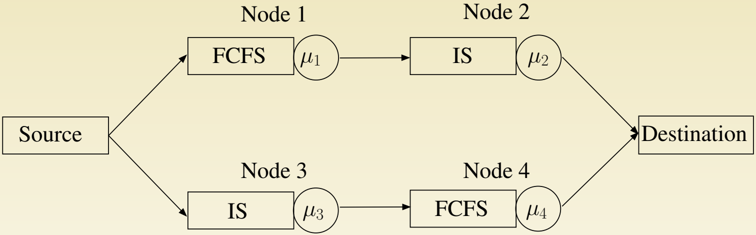
With 3 roads,
$Cost_a = Cost_b = Cost_c = 92$

From the New York Times, Dec 25, 1990, Page 38, What if They Closed 42d Street and Nobody Noticed?, By GINA KOLATA:

" ON Earth Day this year, New York City's Transportation Commissioner decided to close 42d Street, which as every New Yorker knows is always congested. "Many predicted it would be doomsday," said the Commissioner, Lucius J. Riccio. "You didn't need to be a rocket scientist or have a sophisticated computer queuing model to see that this could have been a major problem." But to everyone's surprise, Earth Day generated no historic traffic jam. Traffic flow actually improved when 42d Street was closed. "

Braess Paradoxes: applications

1st example: Cohen-Kelly networks [IKT05]



- Dynamic routing
- Finite number of tasks
- Recurrent equations

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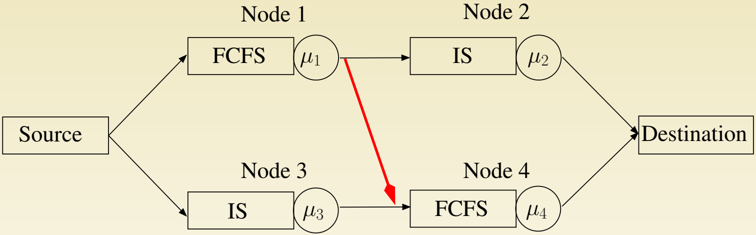

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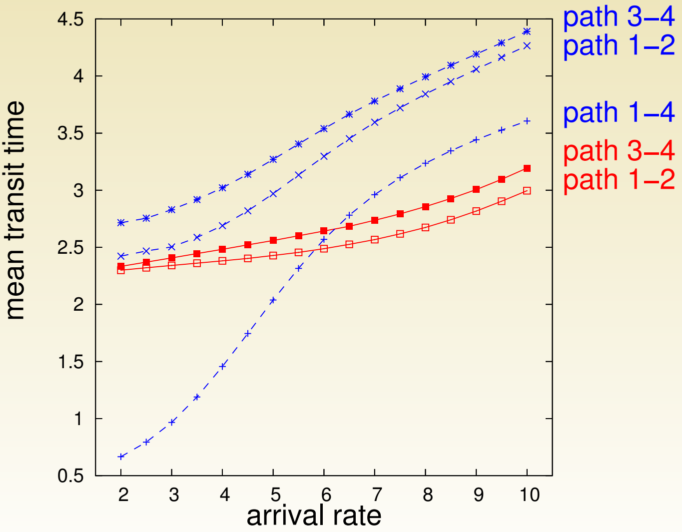
4.5
4
3.5
3
2.5
2
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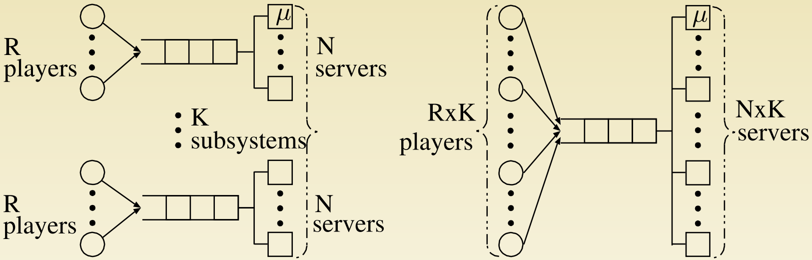
1
0.5
2
3
4
5
6
7
8
9
10
mean transit time
arrival rate
path 3–4
path 1–2
path 1–4
path 3–4
path 1–2

Braess Paradoxes: applications

2nd example: M/M/c queuing systems [IKT06]



- Response time: given by the Erlang formula
- Strategy: choice of arrival rate
- Utility: "power" $\frac{\lambda}{T(\lambda)}$

Braess Paradoxes: applications

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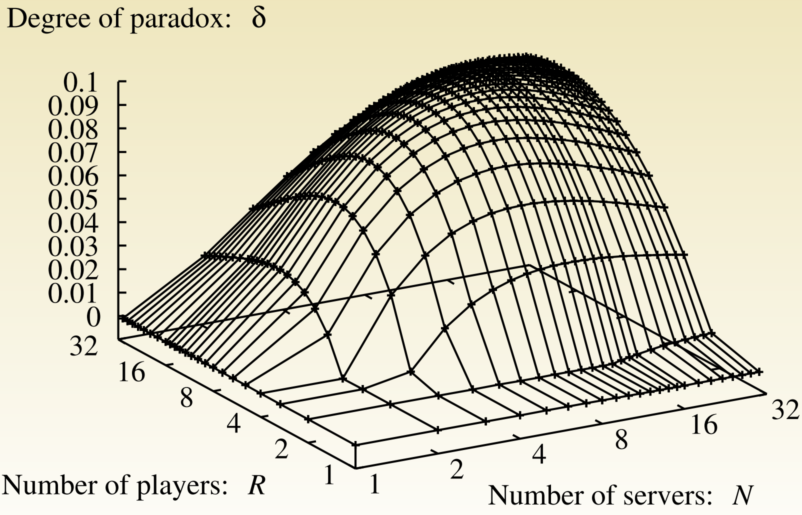
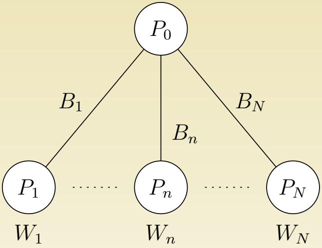




Hypothesis: the master can only send to 1 slave at a time.

Example

maître: $W = 2.55$

3 machines: $(B_i, W_i) = (4.12, 0.41)$, $(4.61, \mathbf{1.31})$, $(3.23, 4.76)$

2 applications: $b^1 = 1$, $w^1 = 2$, $b^2 = 2$, $w^2 = 1$

Equilibrium (ini): $a^1 = 0.173$, $a^2 = 0.0365$

Equilibrium ($W_2 = \mathbf{5.4}$): $a^1 = 0.127$, $a^2 = 0.0168$

Outline

Examples of solutions

No universal solution, but many options:

Correlated equilibria :

- A correlator give advises to each player
- (such that) the optimal strategy for each player is to follow the advice
- Nash equilibria $\subset$ Correlated equilibria

Pricing mechanisms :

- An entity gives money (reward) to players
- Each player strives to maximize its profit

but, implementing mechanisms that ensure optimality is challenging

Be careful - Cuidado!

Do not make a confusion between non-cooperative and distributed (or decentralized), or cooperative and centralized.

The previous examples are decentralized and non-cooperative

TCP is a completely distributed (algorithm) and cooperative (game)

Outline

Axiomatic definition VS optimization problem

1. Pareto optimality
2. Symmetry
3. Invariance towards linear transformations

$+$

- Independant to irrelevant alternatives
  Nash (NBS) / proportional fairness
  $\prod u_i$
- Monotonicity
  Raiffa-Kalai-Smorodinsky / max-min
  Recursively $\max\{u_i | \forall j, u_i \leqslant u_j\}$
- Inverse monotonicity
  Thomson / Social welfare
  $\max \sum u_i$

Fairness family [TAG06]

Introduced by Mo and Walrand

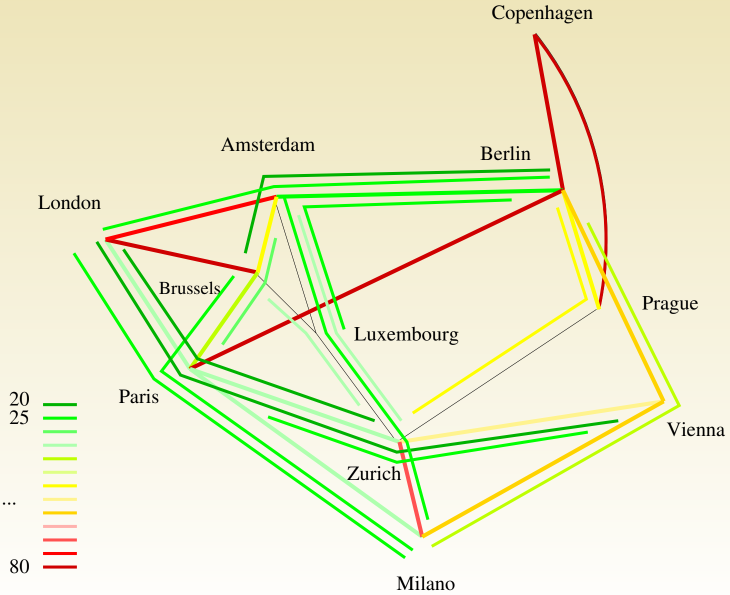
utility — allocation — player

$$\max \sum_{n \in N} \frac{f_n(x_n)^{1-\alpha}}{1-\alpha}$$

Outline

Fairness family: example

The COST network (Prop. fairness)



Fairness family: example

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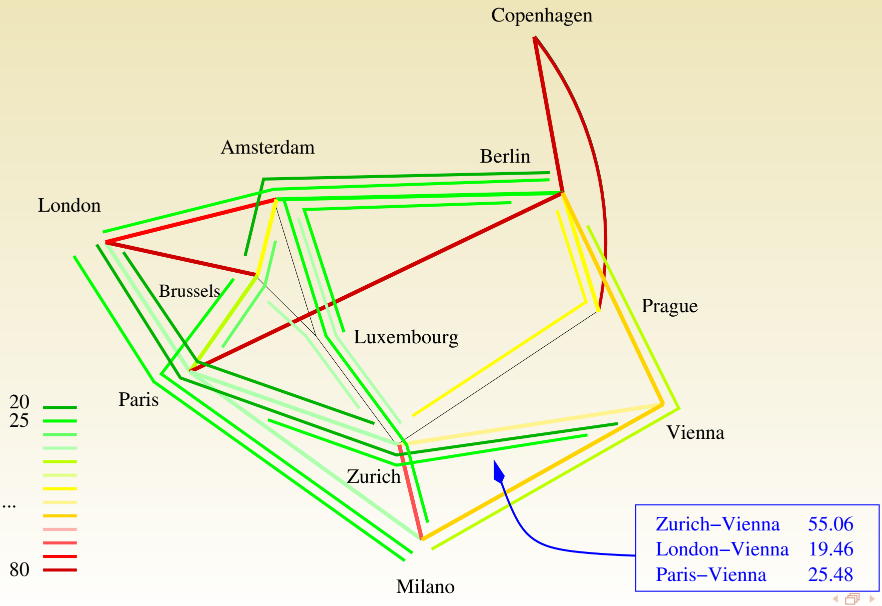
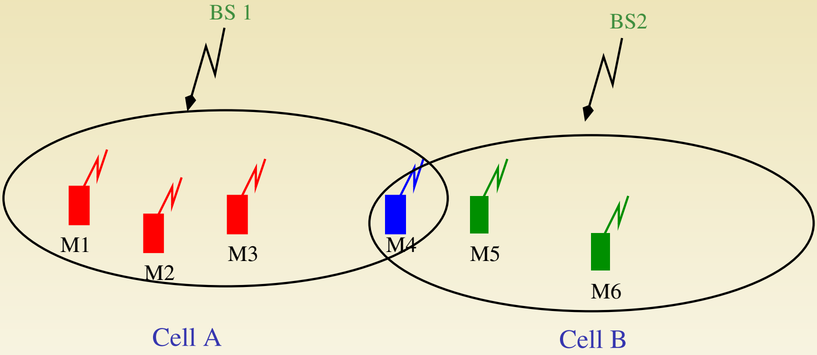


Fairness family: example
CDMA wireless networks [AGT06]



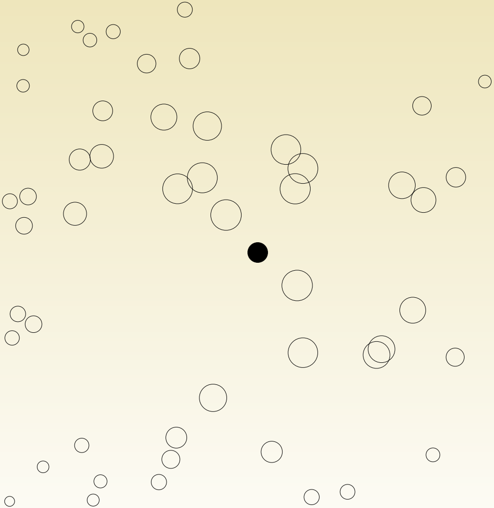
Fair rate allocation: ex. AMR Codec (UMTS) allows 8 rates for voice (between 4.75 and 12.2 kbps) dynamically changed every 20ms.

Model uplink, downlink and macrodiversity

Challenge join allocation of throughput and power

Fairness family: example

CDMA wireless networks [AGT06]



Exemple $\alpha = 0$: global optimization

Fairness family: example

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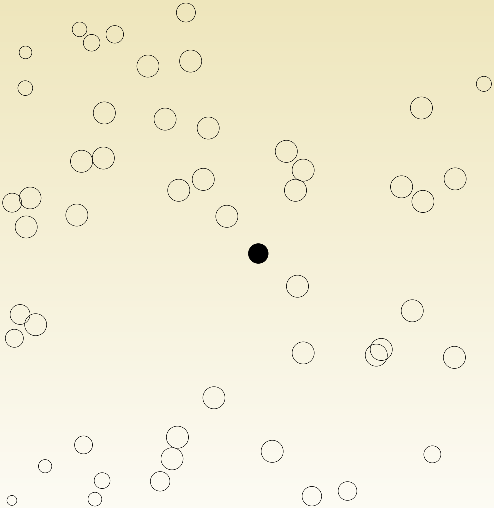


Exemple $\alpha = 2.5$

Fairness family: example

MF-TDMA satellite networks [TAG03]

How to fairly allocate the bandwidth provided by a geostationary satelly among different network operators?

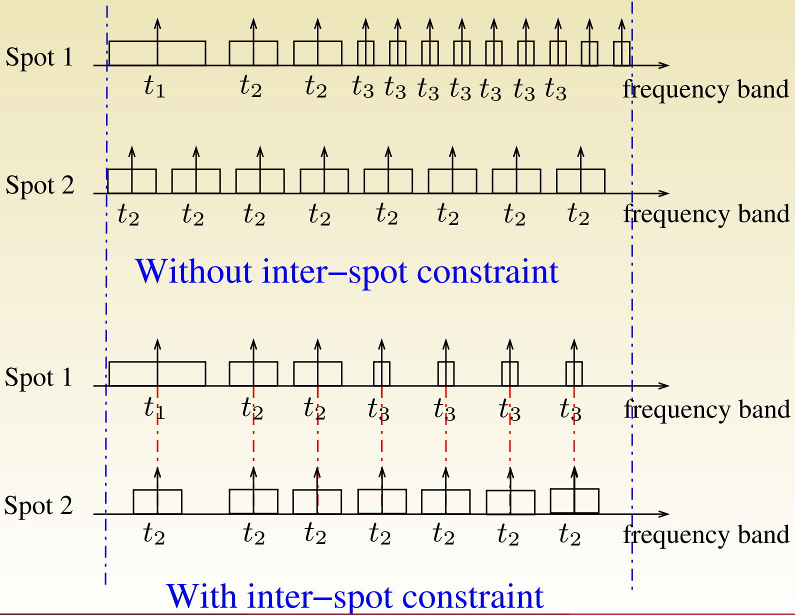
System: MF-TDMA (Multiple Frequency-Time Division Multiple Access), operators ask for a certain number of carriers of certain capacities.

Constraints:

- **Integrity constraints**: N types of carriers, of bandwidth B_1, B_2,...,B_N.
- **Inter-Sopt Compatibility Conditions (ISCC)**:
  - (i) imposing the use of the same frequency plan on ALL spots of a same color
  - (ii) allowing to replace the demand of a client for a carrier j by a carrier t with $t < j$.

Fairness family: example

MF-TDMA satellite networks [TAG03]



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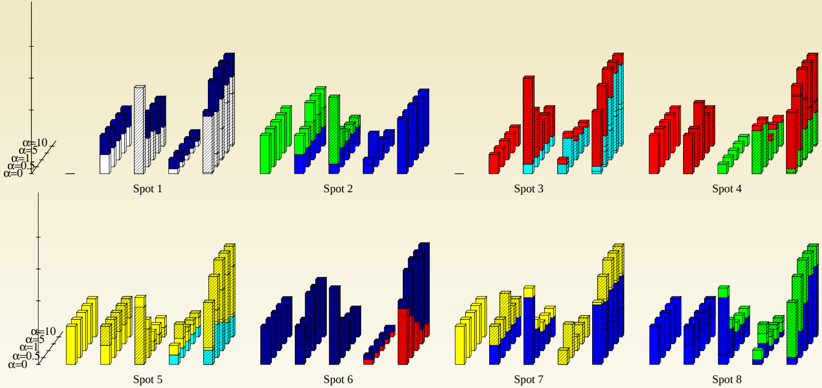
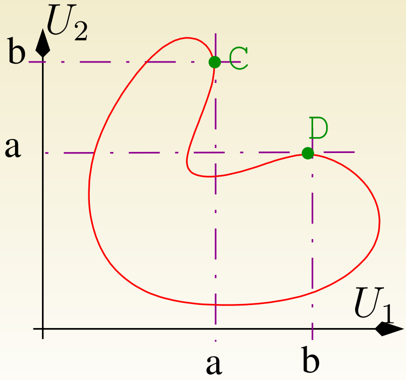


Outline

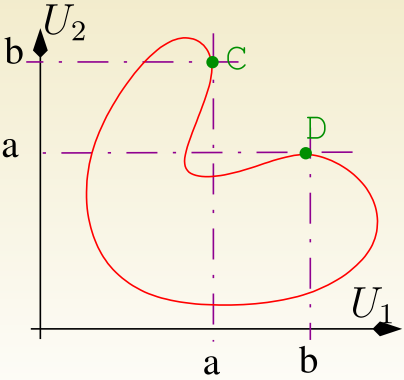
Two points (C and D) can be equally fair (symetrically identital).

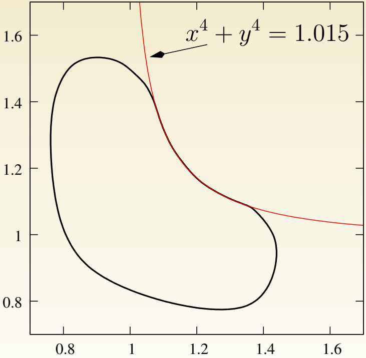


Fairness family: non-convex systems [TKI05]

Two points (C and D) can be equally fair (symetrically identital).

A set of points cannot be differentiated by the α-family.





Conclusion

When multiple users have conflicting objectives cooperation is the way to go to achieve both fairness and efficiency.

But, individual users are proned to act selfishly, which can lead to catastrophic situations (Nash equilibria inefficiencies, Braess paradoxes...).

So, collaboration has to be induced (corelators, pricing mechanisms...) or inforced (penalties).

Conclusion

Example of inforced collaboration (set of rules inforced by the police)

Conclusion

While the purely non-cooperative approach would give...

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