Homogeneous versus hierarchical communication delay model:complexity and approximation

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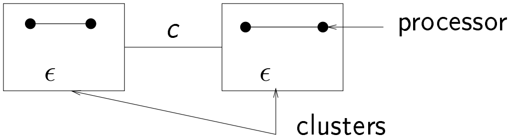
If $c_{ij} = 0, \forall (i,j) \in E$ then we have the model without communication (PRAM model)

An investigation of the scheduling problem from a application on the parallel machine constituted by a set of clusters of processors
To require taking account hierarchical communication
Communications :

- intra-clusters (denoted by ϵ_{ij})
- extra-clusters (denoted by c_{ij})



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- Heterogeneous or Homogeneous processors

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  - $\forall (i, j) \in E, c_{ij} = \epsilon_{ij}$

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- Polynomial-time approximation algorithms are existed according to the type of precedence graph (independent tasks, chains, trees, general precedence graphs) [Trystram,Mounie, Rapine, Dutot]

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- Lower bounds (Negatives results)

Homogeneous model : Negative result (1)

- $\forall i \in V, p_i = 1, \forall (i,j) \in E, c_{ij} = 1$
- Unbounded number of processors

$\nexists$ ρ-approximation with $\rho < \frac{7}{6}$, unless $\mathcal{P} = \mathcal{NP}$
[Hoogeveen,Lenstra,Veltman 94]

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Homogeneous model : Negative result (2)

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- bounded number of processors

$\nexists$ ρ-approximation with $\rho < 1 + \frac{1}{c+3}$, unless $\mathcal{P} = \mathcal{NP}$
[Bampis,Giannakos,König96]

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Hierarchical model : Negative result (1)

- $\forall i \in V, p_i = 1, \forall (i,j) \in E, c_{ij} = 1$ and $\epsilon_{ij} = 0$
- Unbounded number of clusters, 2 identical processors per cluster
- $\bar{P}(P_2)|prec;(c_{ij},\epsilon_{ij}) = (1,0); p_i = 1|C_{max}$

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$\nexists$ ρ-approximation with $\rho < \frac{6}{5}$, unless if $\mathcal{P} = \mathcal{NP}$ [Giroudeau 04]

- $\forall i \in V, p_i = 1, \forall (i,j) \in E, c_{ij} = c$ and $\epsilon_{ij} = c'$ with $c \geq c'$
- Unbounded number of clusters, $m \geq 2$ identical processors per cluster

$\nexists$ ρ-approximation with $\rho < 1 + \frac{1}{c+3}$, unless if $\mathcal{P} = \mathcal{NP}$
[Giroudeau,König 04]

Complexity results

Number of processors	Com. Delay (c_{ij}, ϵ_{ij})	Complexity
$\bar{P}$	$(1,1)$	$\rho \geq \frac{7}{6}$
$\bar{P}$	(c,c)	$\rho \geq \frac{c+5}{c+4}$
P	$(1,1)$	$\rho \geq \frac{4}{3}$
P	(c,c)	$\rho \geq \frac{c+4}{c+3}$
$\bar{P}(P_2)$	$(1,0)$	$\rho \geq \frac{4}{3}$
$\bar{P}(P_2)$	$(c,c'),\ c > c' \geq 1$	$\rho \geq \frac{c+4}{c+3}$
$P(P_2)$ *bipartite*	$(1,1)$	$\rho \geq \frac{4}{3}$

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A lower bound implies that there is non $\mathcal{PTAS}$ for all previous scheduling problems

Homogeneous model : Positive result

- $\exists \frac{4}{3}$-approximation algorithm for the problem $\bar{P}|p_i = 1, c_{ij} = 1|C_{max}$[Munier,König 97]

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- Integer Linear Programming Formulation

$$PL_I \quad \left\{ \begin{array}{lr} & min C_{max} \\ \forall (i,j) \in E, & x_{ij} \in \{0,1\} \\ \forall i \in V, & t_i \geq 0 \\ \forall (i,j) \in E, & t_i + 1 + x_{ij} \leq t_j \\ \forall i \in V - U, & \sum_{j \in \Gamma^+(i)} x_{ij} \geq |\Gamma^+(i)| - 1 \\ \forall i \in V - Z, & \sum_{j \in \Gamma^-(i)} x_{ji} \geq |\Gamma^-(i)| - 1 \\ \forall i \in V & t_i + 1 \leq C_{max} \end{array} \right.$$

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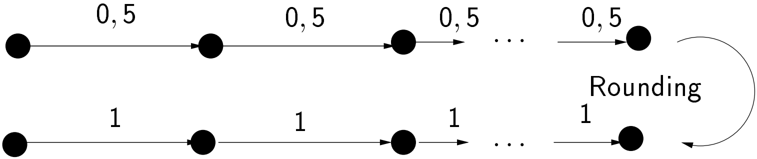
- So,

$$C_{max} \leq \frac{4}{3} C_{max}^{opt}$$

Tightness of the bound

The bound is tight :

a path with k vertices



$C_{max} = 2k - 1$ $\qquad$ $C_{opt} = k + \frac{k-1}{2} = \frac{3k}{2} - \frac{1}{2}$

Dilatation of length by a factor : $\frac{4}{3}$

Hierarchical model : Positive result

- $(2 - \frac{2}{2m+1})$-approximation algorithm for the problem $\bar{P}(P_m)|prec;(c_{ij},\epsilon_{ij}) = (1,0);p_i = 1|C_{max}$ with $m \geq 1$
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- $$\begin{cases} & \min C_{max} \\ \forall (i,j)\in E, & x_{ij}\in\{0,1\} \\ \forall i\in V, & t_i\geq 0 \\ \forall (i,j)\in E, & t_i+1+x_{ij}\leq t_j \\ \forall (i,j)\in E-U, & \sum_{j\in\Gamma^+(i)} x_{ij}\geq |\Gamma^+(i)|-2 \\ \forall (i,j)\in E-Z, & \sum_{j\in\Gamma^-(i)} x_{ji}\geq |\Gamma^-(i)|-2 \\ \forall i,j,k,l,m\in V, & x_{ji}+x_{jk}+x_{lk}+x_{lm}\geq 1 \\ (j,i),(j,k),(l,k),(l,m)\in E, & \\ \forall i,j,k,l,m\in V, & x_{ij}+x_{kj}+x_{kl}+x_{ml}\geq 1 \\ (i,j),(k,j),(k,l),(m,l)\in E, & \\ \forall i\in V, & t_i+1\leq C_{max} \end{cases}$$

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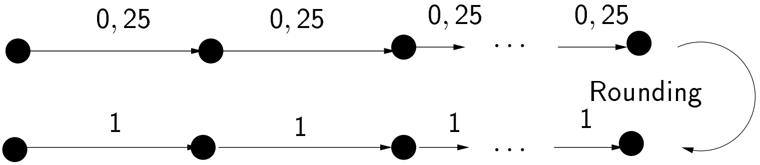
a path with k vertices



$C_{max} \leq 2k$ $C_{opt} = k + \frac{k-1}{4} = \frac{5k}{4} - \frac{1}{4}$

Dilatation of length by $\frac{8}{5}$

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- For the large communication delays $c \geq 2$ (or $(c_{ij} = c, \epsilon_{ij} = c')$ with $c > c' \geq 1$), it exists an-approximation algorithm A such that $A \in \mathcal{APX}$?

Approximation algorithm for Large communication delay in homogeneous model

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- We develop a new algorithm based on the solution given the problem $\bar{P}|prec; c_{ij} = 1; p_i = 1|C_{max}$

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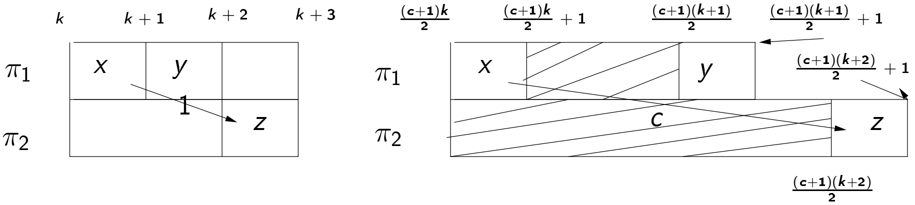
- Third Step An expansion of the makespan, while preserving communication delays ($t_j^c \geq t_i^c + 1 + c$) for i and j with $(i, j) \in E$, processing on two different processors



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- $t_i^c = d \times t_i$ and $t_j^c = d \times t_j$
- After an expansion, $t_j^c \geq t_i^c + 1 + c$, and so $d \times t_i - d \times t_j \geq c + 1, \; d \geq \frac{c+1}{t_i - t_j}, \; d \geq \frac{c+1}{2} \Rightarrow d = \frac{(c+1)}{2}$

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  - $\rho \leq \frac{2(c+1)}{3}$

- The previous algorithm may used to derive a polynomial-time with performance ratio equal to $(c+1)(1-\frac{1}{2m+1})$-approximation algorithm for the problem $\bar{P}(P_m)|prec;(c_{ij},\epsilon_{ij})=(c,c');p_i=1|C_{max}$ with $m \geq 1$

Approximation

# processors	Com. Delay (c_{ij}, ϵ_{ij})	Complexity & Approximation
$\bar{P}$	$(1,1)$	$\frac{7}{6} \leq \rho \leq \frac{4}{3}$
$\bar{P}$	(c,c)	$\frac{c+5}{c+4} \leq \rho \leq \frac{2(c+1)}{3}$
P	$(1,1)$	$\frac{4}{3} \leq \rho \leq \frac{7}{3}$
P	(c,c)	$\frac{c+4}{c+3} \leq \rho \leq \frac{7(c+1)}{6}$
$\bar{P}(P_2)$	$(1,0)$	$\frac{4}{3} \leq \rho \leq \frac{8}{5}$
$\bar{P}(P_2)$	$(c,c'),\ c > c' \geq 1$	$\frac{c+4}{c+3} \leq \rho \leq (c+1)(1-\frac{1}{2m+1})$

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  - There exists a ρ-approximation algorithm with $\rho \in \mathbb{R}$ for the scheduling problem $\bar{P}|p_i = 1, c_{ij} = c \geq 2|C_{max}$?

Conclusion

Any questions ?