Strategies for Replica Placement in Tree Networks

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Introduction and motivation

- Replica placement in tree networks
- Set of clients (tree leaves): requests with QoS constraints, known in advance
- Internal nodes may be provided with a replica;
  in this case they become servers
  and process requests (up to their capacity limit)

How many replicas required?
Which locations?
Total replica cost?

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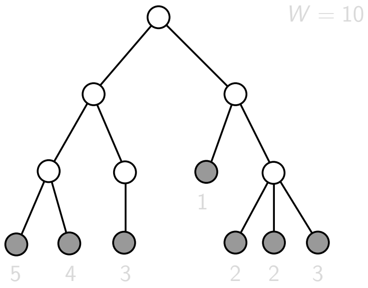
- Handle all client requests, and minimize cost of replicas
- → Replica Placement problem
- Several policies to assign replicas



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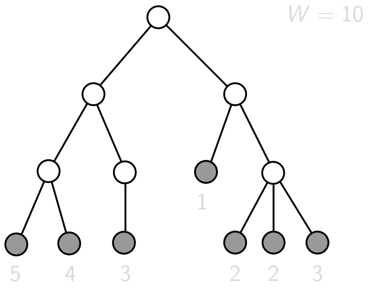

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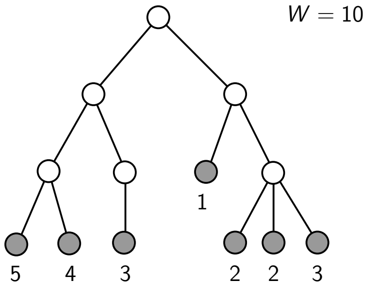

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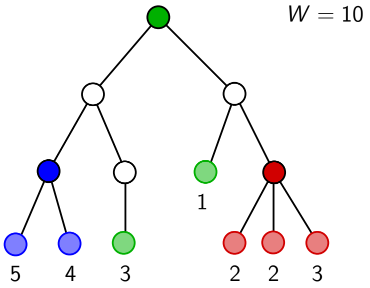

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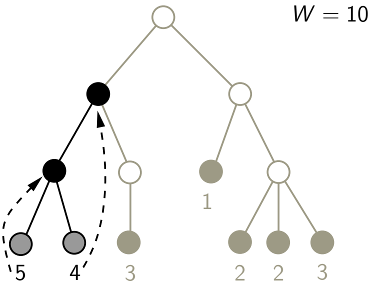

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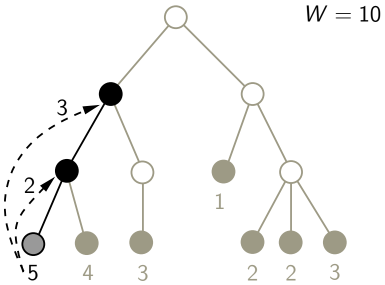

Major contributions

Theory New access policies
Problem complexity
LP-based lower bound to cost of REPLICA PLACEMENT

Practice Heuristics for each policy
Experiments to assess impact of new policies

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Practice
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Outline

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Definitions and notations

- Distribution tree $\mathcal{T}$, clients $\mathcal{C}$ (leaf nodes), internal nodes $\mathcal{N}$
- **Client** $i \in \mathcal{C}$:
  - Sends r_i requests per time unit (number of accesses to a single object database)
  - Quality of service q_i (response time)
- **Node** $j \in \mathcal{N}$:
  - Can contain the object database replica (server) or not
  - Processing capacity W_j
  - Storage cost sc_j
- **Tree edge:** $l \in \mathcal{L}$ (communication link between nodes)
  - Communication time $comm_l$
  - Bandwidth limit BW_l

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Tree notations

- r: tree root
- children(j): set of children of node $j \in \mathcal{N}$
- parent(k): parent in the tree of node $k \in \mathcal{N} \cup \mathcal{C}$
- link $l : k \rightarrow \text{parent}(k) = k'$. Then succ($l$) is the link $k' \rightarrow \text{parent}(k')$ (when it exists)
- Ancestors(k): set of ancestors of node k
- If $k' \in \text{Ancestors}(k)$, then path$[k \rightarrow k']$: set of links in the path from k to k'
- subtree(k): subtree rooted in k, including k.

Problem instances

- Goal: place replicas to process client requests
- Client $i \in \mathcal{C}$: Servers(i) $\subseteq \mathcal{N}$ set of servers responsible for processing its requests
- $r_{i,s}$: number of requests from client i processed by server s ($\sum_{s \in \text{Servers}(i)} r_{i,s} = r_i$)
- $R = \{s \in \mathcal{N} | \ \exists i \in C \ , \ s \in \text{Servers}(i)\}$: set of replicas

Constraints

- **Server capacity** – $\forall s \in R, \sum_{i \in \mathcal{C} | s \in \text{Servers}(i)} r_{i,s} \leq \text{W}_s$
- **QoS** – $\forall i \in \mathcal{C}, \forall s \in \text{Servers}(i), \sum_{l \in \text{path}[i \to s]} \text{comm}_l \leq \text{q}_i$.
- **Link capacity** – $\forall l \in \mathcal{L} \sum_{i \in \mathcal{C}, s \in \text{Servers}(i) | l \in \text{path}[i \to s]} r_{i,s} \leq \text{BW}_l$

Objective function

- Min $\sum_{s \in R} sc_s$
- Restrict to case where $sc_s = W_s$
- REPLICA COST problem: no QoS nor bandwidth constraints; heterogeneous servers
- REPLICA COUNTING problem: idem, but homogeneous platforms

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Single server vs Multiple servers

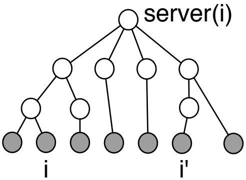
Single server – Each client i is assigned a single server server(i), that is responsible for processing all its requests.

Multiple servers – A client i may be assigned several servers in a set Servers(i). Each server $s \in$ Servers(i) will handle a fraction $r_{i,s}$ of the requests.

In the literature: single server policy with additional constraint.

Closest policy

- *Closest*: single server policy
- Server of client i is constrained to be first server found on the path that goes from i upwards to the tree root
- Consider a client i and its server server(i):
  $\forall i' \in \text{subtree}(\text{server}(i)), \quad \text{server}(i') \in \text{subtree}(\text{server}(i))$
- Requests from i' cannot "traverse" server(i) and be served higher



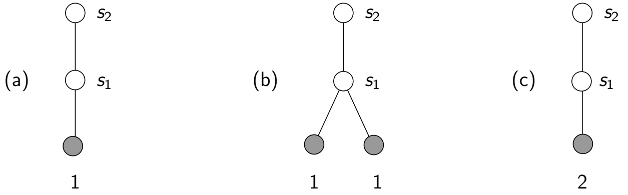
Upwards and *Multiple* policy

- New policies not studied in the literature

- *Upwards*: *Closest* constraint is relaxed
- *Multiple*: relax single server restriction

- Expect more solutions with new policies, at a lower cost
- QoS constraints may lower difference between policies

Example: existence of a solution



$W = 1$

- (a): solution for all policies
- (b): no solution with *Closest*
- (c): no solution with *Closest* nor *Upwards*

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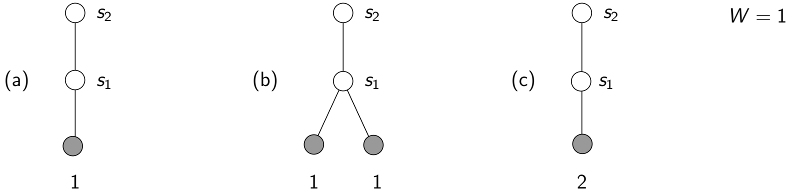

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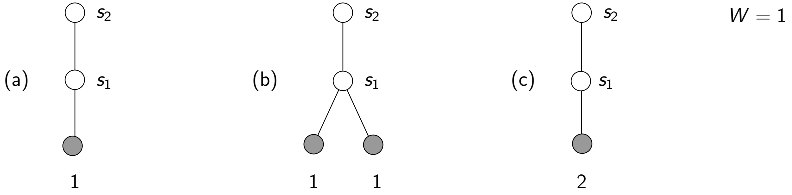

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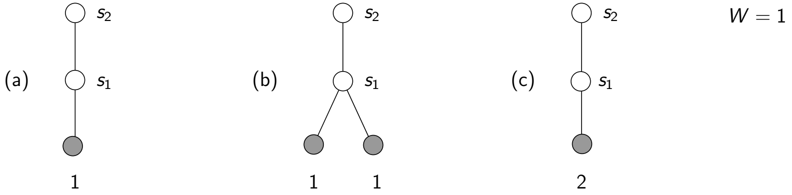

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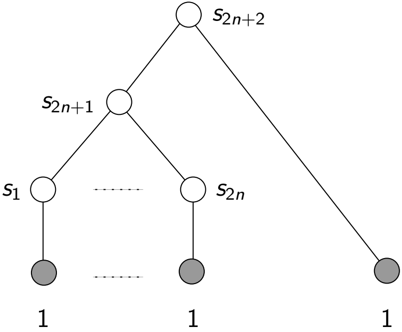
Upwards versus *Closest*

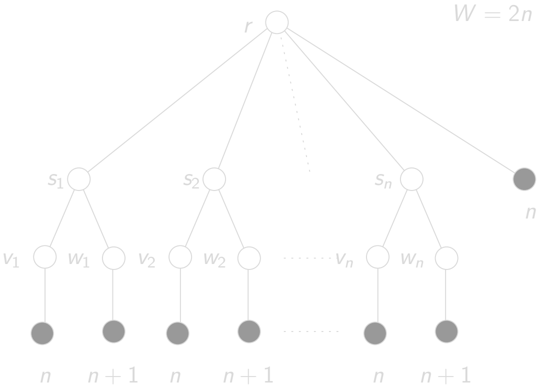


$W = n$

- *Upwards*: 3 replicas in s_{2n}, s_{2n+1} and s_{2n+2}
- *Closest*: at least $n+2$ replicas (replica in s_{2n+1} or not)

Multiple versus *Upwards*

- Replica Counting: *Multiple* twice better than *Upwards*.
- Performance ratio: open problem.



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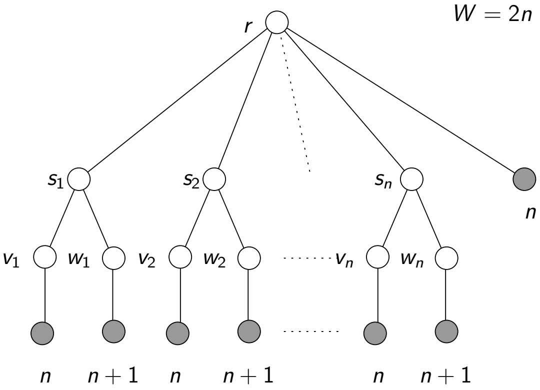

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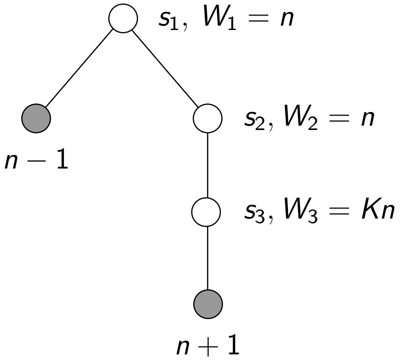
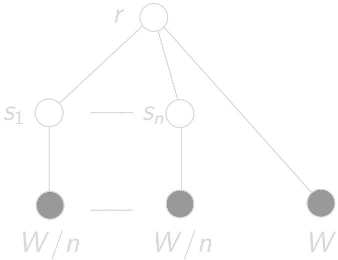
- Replica Cost: *Multiple* arbitrarily better than *Upwards*



Multiple: cost $2n$ / *Upwards*: cost $(K+1)n$

Lower bound for the REPLICA COUNTING problem

Obvious lower bound: $\left\lceil \frac{\sum_{i \in \mathcal{C}} r_i}{W} \right\rceil = 2$



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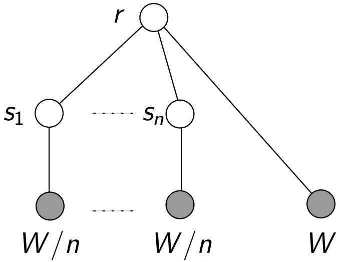

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Complexity results - Basic problem

	Replica Counting **Homogeneous**	Replica Cost **Heterogeneous**
Closest **Upwards** **Multiple**	polynomial [Cidon02,Liu06]	

Table: Complexity results for the different instances of the problem

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- All instances for the Heterogeneous case are NP-complete

Complexity results - QoS and Bandwidth

- *Closest*/Homogeneous + QoS: Polynomial (Liu et al.)
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Multiple/Homogeneous: greedy algorithm

3-pass algorithm:

- Select nodes which can handle W requests
- Select some extra servers to fulfill remaining requests
- Decide which requests are processed where

Example to illustrate algorithm (informally)

Proof of optimality: any optimal solution can be transformed into a solution similar to the one of the algorithm (moving requests from one server to another)

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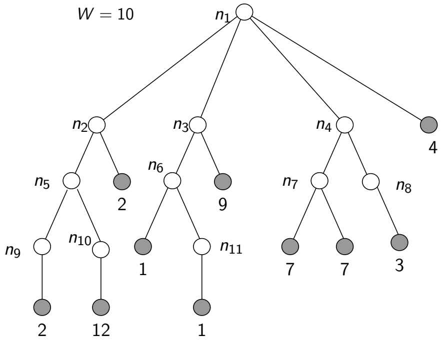

Initial network

The example network

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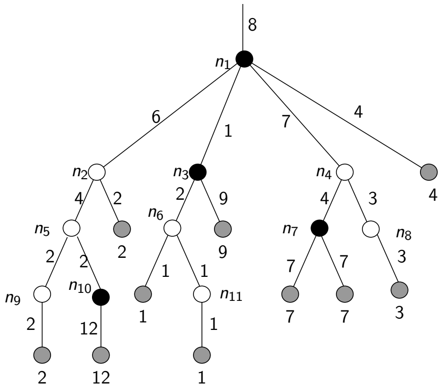

Pass 1

Placing *saturated* replicas

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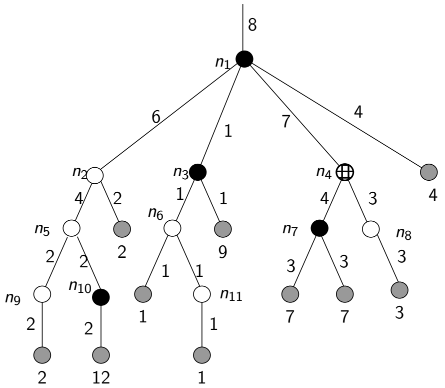

Pass 2

Placing extra replicas: n_4 has maximum useful flow

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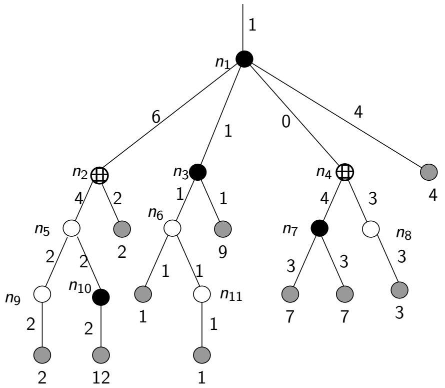

Pass 2

Placing extra replicas: n_2 is of maximum useful flow 1

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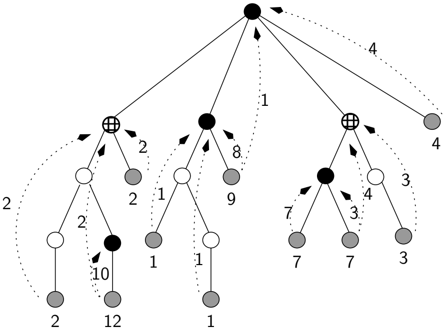
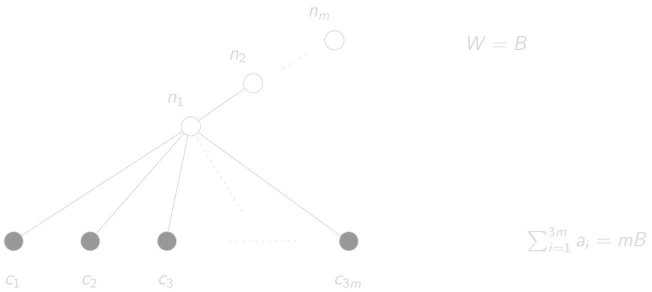


Pass 3

Deciding where requests are processed

Upwards/Homogeneous

- The REPLICA COUNTING problem with the *Upwards* strategy is NP-complete in the strong sense
- Reduction from 3-PARTITION



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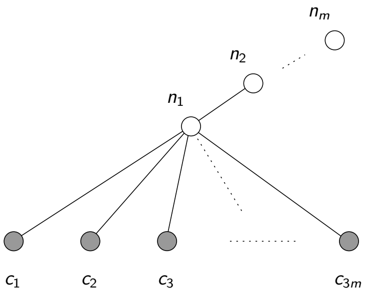
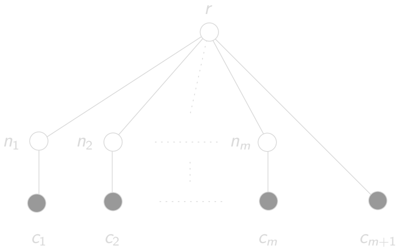


$W = B$

$\sum_{i=1}^{3m} a_i = mB$

Heterogeneous network: Replica Cost problem

- All three instances of the Replica Cost problem with heterogeneous nodes are NP-complete
- Reduction from 2-PARTITION



$\sum_{i=1}^{m} a_i = S$, $a_{m+1} = 1$, $W_j = a_i$, $W_r = S/2 + 1$

Solution with total storage cost $S + 1$?

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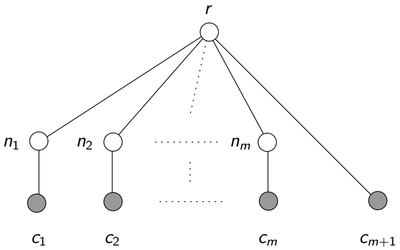

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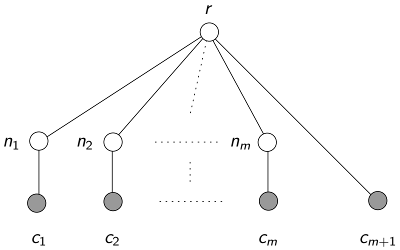

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Linear programming

- General instance of the problem
  - Heterogeneous tree
  - QoS and bandwidth constraints
  - *Closest*, *Upwards* and *Multiple* policies
- Integer linear program: no efficient algorithm
- Absolute lower bound if program solved over the rationals (using the GLPK software)
- *Closest*/*Upwards* LP formulation

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- x_j: boolean variable equal to 1 if j is a server (for one or several clients)
- $y_{i,j}$: boolean variable equal to 1 if $j = \text{server}(i)$
  - If $j \notin Ancests(i)$, $y_{i,j} = 0$
- $z_{i,l}$: boolean variable equal to 1 if link $l \in \text{path}[i \to r]$ used when i accesses server(i)
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Objective function: $\sum_{j \in \mathcal{N}} \text{sc}_j x_j$

Linear program: variables

- x_j: boolean variable equal to 1 if j is a server (for one or several clients)
- $y_{i,j}$: boolean variable equal to 1 if $j = \text{server}(i)$
  - If $j \notin Ancests(i)$, $y_{i,j} = 0$
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Linear program: constraints

- Servers: $\forall i \in \mathcal{C}, \sum_{j \in \text{Ancestors}(i)} y_{i,j} = 1$
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Multiple formulation

Multiple

- Similar formulation, with
  - $y_{i,j}$: integer variable = nb requests from client i processed by node j
  - $z_{i,l}$: integer variable = nb requests flowing through link l
- Constraints are slightly modified

A mixed integer LP-based optimal solution

- Solving over the rationals: solution for all practical values of the problem size
  - Not very precise bound
  - *Upwards*/*Closest* equivalent to *Multiple* when solved over the rationals
- Integer solving: limitation to $s \leq 50$ nodes and clients
- Mixed bound obtained by solving the *Upwards* formulation over the rational and imposing only the x_j being integers
  - Resolution for problem sizes $s \leq 400$
  - Improved bound: if a server is used only at 50% of its capacity, the cost of placing a replica at this node is not halved as it would be with $x_j = 0.5$.
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Outline

Heuristics

- Polynomial heuristics for REPLICA COST problem
  - Heterogeneous platforms
  - No bandwidth constraints
  - Heuristics with and without QoS
- Experimental assessment of relative performance of the three policies
- Impact of QoS
- Traversals of the tree, bottom-up or top-down
- Worst case complexity $O(s^2)$, where $s = |\mathcal{C}| + |\mathcal{N}|$ is problem size

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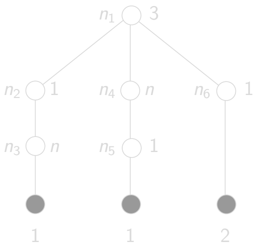
Heuristics for *Closest*

- Closest Top Down All **CTDA**
  - Breadth-first traversal of the tree
  - When a node can process the requests of all the clients in its subtree, node chosen as a server and exploration of the subtree stopped
  - Procedure called until no more servers are added
  - Choosing n_2, n_4 and then n_1



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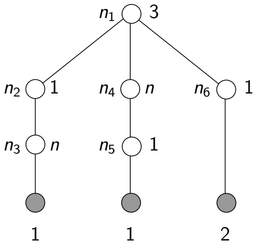

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Heuristics for *Closest*

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  - Traversal of the tree, treating subtrees that contains most requests first
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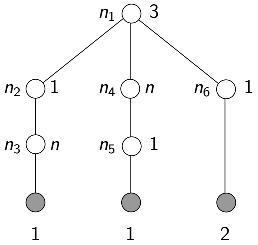

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Heuristics for *Closest*

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- Closest Bottom Up **CBU**
  - Bottom-up traversal of the tree
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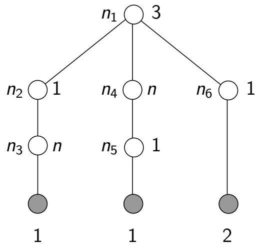
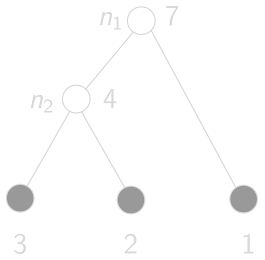
  - Choosing n_3, n_5, n_1



Heuristics for *Upwards*

- Upwards Top Down **UTD**
  - 2-pass algorithm
  - Select first saturating nodes, then extra nodes
  - Choosing n_2 (for c_1) and in second pass n_1 (for c_2, c_3)



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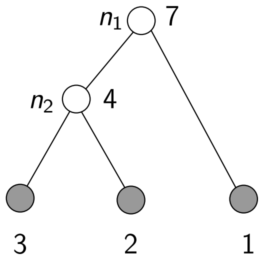

Heuristics for *Upwards*

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Heuristics for *Upwards*

- Upwards Top Down **UTD**
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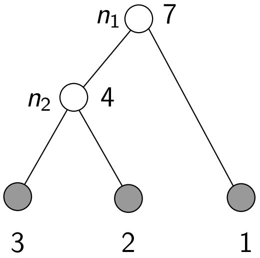
  - Sorting clients by decreasing request numbers, and finding the server of minimal available capacity to process its requests.
  - Choosing n_2 for c_1, n_1 for c_2 and n_1 for c_3



Heuristics for *Multiple*

- A top-down and a bottom-up heuristic in 2-passes (**MTD**, **MBU**)
- A greedy heuristic **MG**, similar to Pass 3 of the polynomial algorithm for *Multiple*/Homogeneous: fill all servers as much as possible in a bottom-up fashion

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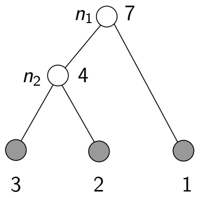

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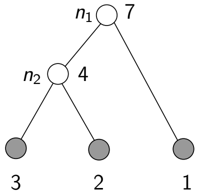

  - MG affects 4 requests to n_2, and then the remaining 2 requests to n_1
  - CTDLF better on this example: selects n_1 only

Heuristics for *Multiple*

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- A greedy heuristic **MG**, similar to Pass 3 of the polynomial algorithm for *Multiple*/Homogeneous: fill all servers as much as possible in a bottom-up fashion
- Heuristic MixedBest **MB** which picks up best result over all heuristics: solution for the *Multiple* policy

Heuristics with QoS

- Bunch of similar polynomial heuristics with QoS constraints
- Tradeoff between big and QoS-critic clients
- Identifying indispensable servers: clients with QoS=1 need be served by their parent, and so on
- MixedBest heuristic which picks up the best result

Outline

Plan of experiments

- Assess impact of the different access policies
- Assess performance of the polynomial heuristics
- Assess impact of QoS

- Important parameter:

$$\lambda = \frac{\sum_{i \in \mathcal{C}} r_i}{\sum_{j \in \mathcal{N}} W_j}$$

- 30 trees for each $\lambda = 0.1, 0.2, ..., 0.9$
- Problem size $s = |\mathcal{C}| + |\mathcal{N}|$ such that $15 \leq s \leq 400$
- Computation of the LP optimal solution for each tree

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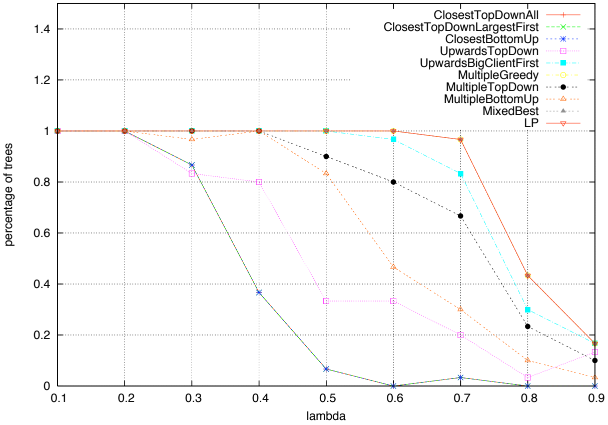
Results - Percentage of success

- Number of solutions for each lambda and each heuristic
- No LP solution → No solution for any heuristic
- Homogeneous case



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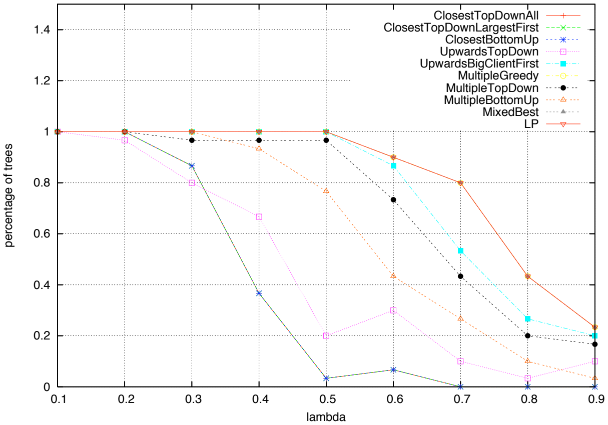

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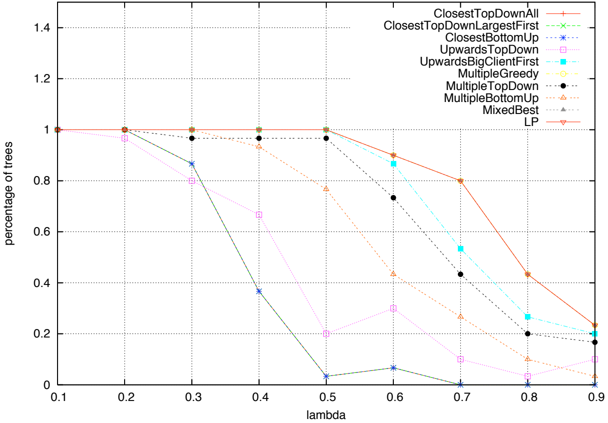

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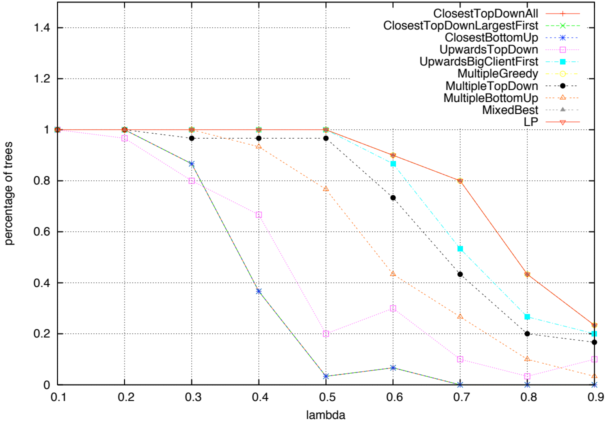

- Striking impact of new policies
- MG and MB always find the solution

Results - Solution cost

- Distance of the result (in terms of replica cost) of the heuristic to the optimal
- T_λ: subset of trees with a solution
- Relative cost:

$$rcost = \frac{1}{|T_\lambda|} \sum_{t \in T_\lambda} \frac{cost_{LP}(t)}{cost_h(t)}$$

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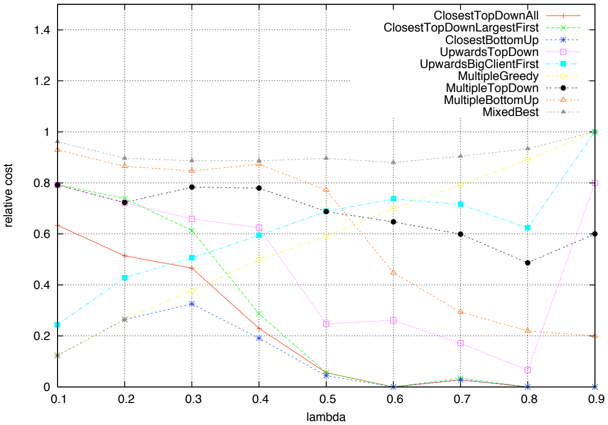
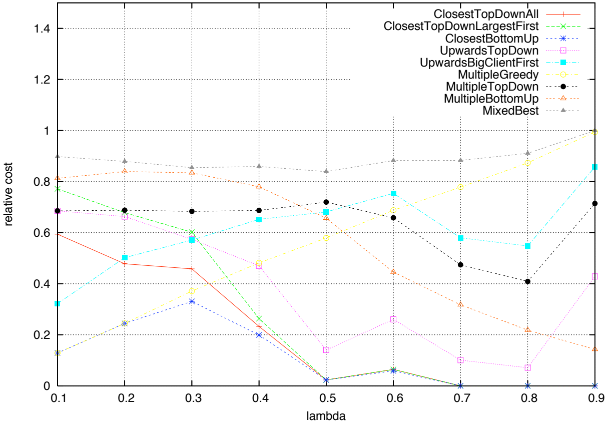


Results - Solution cost

- Heterogeneous results - similar to the homogeneous case

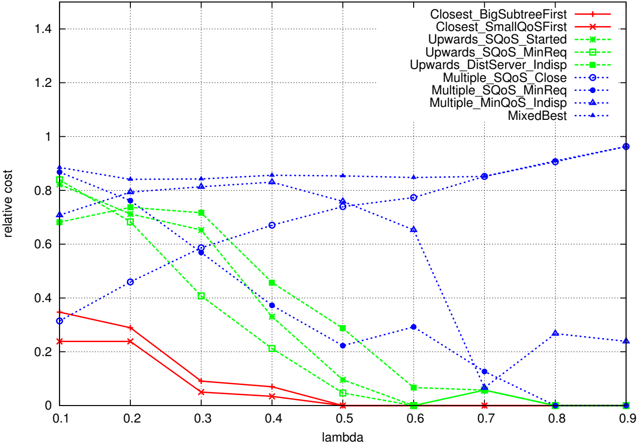


Summary

- Striking effect of new policies: many more solutions to the Replica Placement problem
- *Multiple* $\geq$ *Upwards* $\geq$ *Closest*: hierarchy observed within our heuristics
- Best *Multiple* heuristic (MB) always at 85% of the optimal: satisfactory result

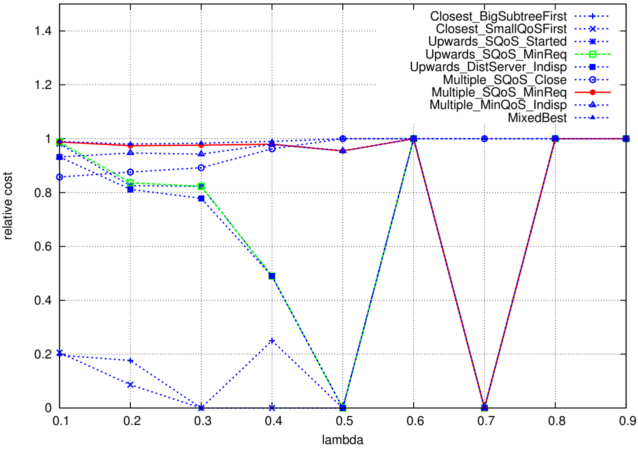
Results - QoS impact

Big trees, $average(QoS) = height/2$



Results - QoS impact

Big trees, $QoS \in \{1, 2\}$ - very constrained



Summary

- *Multiple* $\geq$ *Upwards* $\geq$ *Closest*: hierarchy also under QoS-constraints
- Best *Multiple* heuristic (MB) at 80% of optimal solution with average QoS constraints: $average(qos) = height/2$
- Better results with big trees (height between 16 and 21) than smaller trees

$QoS \in \{1, 2\}$:	95% (vs 90% with an exception)
$average(QoS) = height/2$:	85% (vs 80%)
no QoS:	85% (vs 70%)

- Good performance of heuristics with QoS

Outline

Extensions

- Simplified problem instance for this work
- Possible generalizations:
  - Several objects
  - More complex objective function

Extensions - Several objects

- We considered a single object: all replicas are identical
- Different types of objects need to be accessed: clients have requests of different types
- New parameters:
  - Requests per object r_i^k, and $q_i^{(k)}$
  - Size of the object, computation time involved, storage cost, ...
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Cost of replica – What we considered in this work

Communication cost – This cost is the *read* cost

Update cost – The *write* cost is the extra cost due to an update of the replicas

Linear combination – A quite general objective function can be obtained by a linear combination of the three different costs

$$\alpha \sum_{\text{servers, objects}} \textit{replica cost} + \beta \sum_{\text{requests}} \textit{read cost} + \gamma \sum_{\text{updates}} \textit{write cost}$$

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Outline

Related work

- Several papers on replica placement, but...
- ...all consider only the *Closest* policy
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- Wolfson and Milo: impact of the *write* cost, use of a minimum spanning tree for updates. Tree networks: polynomial solution
- Cidon et al (multiple objects) and Liu et al (QoS constraints): polynomial algorithms for homogeneous networks.
- Kalpakis et al: NP-completeness of a variant with bidirectional links (requests served by any node in the tree)
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Introduction of two new policies for the REPLICA PLACEMENT problem, *Upwards* and *Multiple*: natural variants of the standard *Closest* approach → surprising they have not already been considered

Theoretical side – Complexity of each policy, for homogeneous and heterogeneous platforms

Practical side

- Design of several heuristics for each policy
- Comparison of their performance
- Striking impact of the policy on the result
- QoS is not changing the hierarchy of policies
- Use of a LP-based optimal solution to assess the absolute performance, which turns out to be quite good.

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Future work

Short term

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- Designing heuristics for more general instances of the REPLICA PLACEMENT problem (bandwidth constraints): these constraints may lower the difference between policies

Longer term

- Consider the problem with several object types
- Extension with more complex objective functions

Still a lot of challenging algorithmic problems ☺

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